Letter

Air resistance of spherical fireworks shells

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Abstract
Free falling and wind tunnel experiments of model spherical fireworks shells have been carried out, in order to examine the effect of air resistance to the exterior ballistics of the shells. It was found that at the initial stage of free falling the air resistance is negligible and the result of ballistic calculation is practically not affected by using different drag coefficients. The drag coefficients of empty shells were estimated by free falling and wind tunnel experiments at a *Re* region.

Keywords: Fireworks, Model shell, Free falling, Wind tunnel, Air resistance

1. Introduction

Japanese spherical fireworks shell is shot from a mortar, bursts after reaching to the maximum height, expels burning stars, and the burning stars express various artificial flowers. These phenomena can be divided into the behavior of the shell in the mortar and in the air, the bursting of the shell, and the movement of the ejected burning stars. In this article, we describe results of the experiments for examining the behaviors of the shell in the air, especially of the air resistance of the shell observed by free falling and wind tunnel experiments.

There are pioneering works by Shimizu[1), 2)] on the exterior ballistics of spherical fireworks shells. Experiment for estimating scatter of black shell or 'dud' shell was carried out and the obtained data were analyzed by Matsunaga *et al.*[3)]. Recently, Hatanaka[4)] published a similar work. These works measured altitude attained, time of flight, descent of 'dud' shell, and initial velocity of the shell. From these data, drag coefficient C_D was estimated.

J. E. Mercer[6)] developed a computer code that numerically models fireworks ballistics, including the gas dynamics of black powder combustion, leakage flow around the shell and aerodynamics of flight. M. S. Russell[7)] gave a short lecture on the ballistics of shells in his book.

In this study, we tried to know the nature of drag coefficient for the fireworks shells from free falling and wind tunnel experiments.

2. Experimental

2.1 Materials

Model empty aerial shells were provided with by Sunaga Fireworks Company. Dimensions of the shells are listed in Table 1.

2.2 Free falling experiment of empty shells in the short distance

The short distance falling experiment was carried out in the corridor of a building using a high speed video camera (FOR.A VFC-100SB) and a vertical measuring pole of 2 m height. An empty shell was released from the top of the pole and the motion of the free falling shell was recorded by the camera.

Table 1 Diamter and mass of empty shells.

No.	2	2.5	3	4	5	7	10
Diameter (m)	0.057	0.069	0.084	0.112	0.142	0.197	0.272
Mass (kg)	0.024	0.044	0.070	0.141	0.209	0.477	0.920

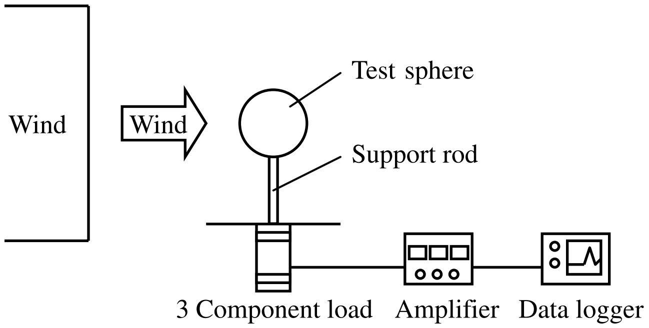

Fig. 1 Wind tunnel and drag coefficient measuring system.

2.3 Free falling experiment in a tower

Next free falling experiment was done in an emergency staircase of about 20 m height. The passing times of the shell at definite 4 points were determined using 4 high speed video cameras (FOR.A VFC-100SB and PHOTRON FASTCAM-Rabbit). The 4 cameras started simultaneously by a trigger.

2.4 Wind tunnel experiment

The drag coefficient C_D of No. 5 shell (0.142 m in diameter) was determined using an open-circuit type wind tunnel. The opening area of the wind tunnel is 1.05 m × 1.05 m and wind speed can be changed from 1 $m \cdot s^{-1}$ to 20 $m \cdot s^{-1}$. The wind tunnel and drag coefficient measuring system are shown in Fig. 1.

The test sphere is supported by a support rod in the wind tunnel. Drag D_{S+R} for the test sphere and the support rod are measured using the 3-component load cell (Nissyo Electric Co.LMC-3502) in each wind speed. Then, D_R for the rod is measured similarly, and D_S for the sample is obtained by subtracting D_R from D_{S+R}.

The drag coefficient C_D of the test sphere is calculated from D_S using Equation (1).

$$C_D = \frac{D_S}{\frac{1}{2}\rho\frac{1}{4}\pi d^2 U_\infty^2} \tag{1}$$

Here, ρ, d, and U_∞ are density and diameter of the test sphere, and the wind velocity of the wind tunnel.

3. Results and Discussion

3.1 Effect of air resistance on the shell in the short distance

It is known that the motion of a sphere in the air is affected by air resistance. The motion of a sphere in the air is expressed by Equation (2).

$$\frac{du}{dt} = g - \frac{3\rho_f}{4\rho D} C_D \cdot u^2 \tag{2}$$

Here, u, t, g, ρ_f, ρ, D and C_D are velocity of a sphere, time, acceleration of gravity, air density, density of the sphere, diameter of the sphere, and drag coefficient of the air.

Assuming that C_D is constant, the falling time (t) and the falling height (h) is expressed by Equations (3) and (4), respectively.

$$t = \frac{1}{2ab}\ln\left(\frac{a+bu}{a-bu}\right) \tag{3}$$

$$h = -\frac{1}{2b^2}\ln\left(\frac{a^2-b^2u^2}{a^2}\right) \tag{4}$$

Here, $a^2 = g$ and $b^2 = \frac{3\rho_f}{4\rho D}C_D$, respectively.

Assuming constant drag coefficients, falling time and falling distance of a No. 2 empty shell were calculated and their relationships shown in Fig. 2.

From Fig. 2, it is found that the relationship between falling time (t) and falling height (h) is not dependent on C_D when t and h is small. In this area, the relationship between t and h can be expressed by neglecting air resistance or using any small constant drag coefficient.

Table 2 Falling height (H), time (t) and standard deviation (std) for No. 2 empty shell.

H (m)	Corrected t_{obs} (s)					t_{calc}	
Exp. No.	$t1$	$t2$	$t3$	$t4$	Average		std
0.0	0	0	0	0	0	0	
0.2	0.202	0.202	0.202	0.202	0.202	0.202	
0.4	0.282	0.286	0.290	0.294	0.288	0.286	0.0058
0.6	0.346	0.350	0.354	0.354	0.351	0.350	0.0040
0.8	0.406	0.402	0.410	0.414	0.408	0.404	0.0069
1.0	0.450	0.450	0.454	0.458	0.453	0.452	0.0041
1.2	0.494	0.494	0.498	0.502	0.497	0.495	0.0046
1.4	0.534	0.534	0.538	0.542	0.537	0.535	0.0048
1.6	0.574	0.570	0.578	0.582	0.576	0.571	0.0074
1.8	0.610	0.606	0.610	0.618	0.611	0.606	0.0076
Average							0.0045

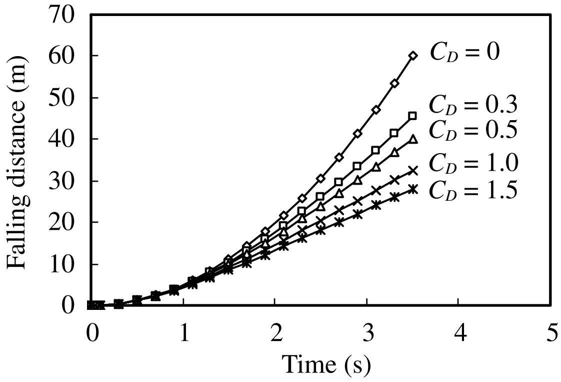

Fig. 2 Plot of calculated falling distance vs. time for No. 2 shell.

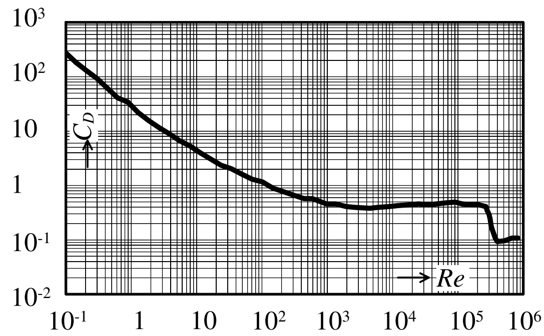

Fig. 3 Plot of C_D vs. Re[5)].

It is difficult to determine exactly the start time of free falling. We determined the start time of free falling of a model empty shell from the short distance free falling experiment and the Newtonian motion equation without air resistance.

In our experiments, an empty shell is released from a clip and at the same time a trigger signal sent to the video camera. Then the camera starts. The camera records the times at definite distances of the shell from the start point.

Table 3 Observed and calculated falling times (t_{calc} and t_{obs}), and standard deviation (*std*) for No. 2 empty shell when C_D =0.7.

H (m)	T_{obs} (s)	T_{calc} (s)
4.30	0.994	0.967
4.40	1.006	0.979
7.90	1.367	1.346
8.00	1.375	1.355
11.50	1.673	1.666
11.60	1.681	1.675
11.70	1.690	1.683
15.24	1.954	1.970
15.33	1.969	1.977
22.40	2.456	2.507
std		0.024

Air density = 1.205 kg·m^{-3};
viscosity coefficient of air = 1.81×10^{-5} Pa·s
kinematical viscosity coefficient of air = 1.50×10^{-5} m^2·s^{-1}
Shell No. 2: mass = 0.024 kg, diameter = 0.057 m, density = 247.5 kg·m^{-3}

The start time of the shell is not necessarily correct in this experiment. The start time was corrected by the observed time at 0.2 m lower than the start point using the free falling motion equation of the shell without air resistance.

The corrected falling times and standard deviations for a No. 2 shell are listed in Table 2.

Averaged standard deviation was 0.0045 sec. This is comparable to the accuracy of the high speed camera 0.004 sec·frame^{-1}. These data indicate that in the short falling distance the motion of the empty shell obeys the free falling equation without air resistance.

For No. 2.5, 3, 4 and 5 empty shells, the averaged standard deviations of the falling heights were 0.0036, 0.0019, 0.0033 and 0.0073 m, respectively.

3.2 Free falling experiment in a tower

The drag coefficient C_D of a sphere is known as a function of Reynolds Number (Re) and shown in Fig. 3[5)].

For small Re, C_D decreases with Re and then nearly constant until a critical Re. At the critical Re, C_D decreases with Re sharply to a smaller nearly constant value.

For small Re any constant small C_D can be used as discussed earlier, so constant C_D can be used for Re smaller than critical value in the free falling of fireworks shells.

The free falling experiment was carried out in a tower using No. 2, 4 and 5 empty shells. The falling times were observed at from 4.3 m to 22.4 m lower than the start point. Assuming several C_D s, corresponding falling times and standard deviations were calculated. An example of observed and calculated data is listed in Table 3.

Table 4 Assumed C_D and calculated std.

No. \ C_D	0	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
2	0.172	0.096	0.072	0.048	0.028	<u>0.024</u>	0.042	0.066	0.091
4	0.119	0.072	0.056	0.041	0.027	<u>0.020</u>	0.024	0.037	0.051
5	0.146	0.094	0.077	0.059	0.043	0.027	<u>0.017</u>	0.023	0.037

The standard deviations of calculated falling times from observed ones are listed in Table 4 for No. 2, 4 and 5 empty shells.

The standard deviations were least when C_D = 0.7, 0.7 and 0.8 for No. 2, 4 and 5 shells, respectively. The corresponding mean *Re* were 4.3×10^4, 8.9×10^4 and 11.0×10^4, respectively.

3.3 Wind tunnel experiment

Wind - tunnel experiment was carried out for a No. 5 empty shell. Results are shown in Fig. 4.

Observed C_D was 0.4 for the No. 5 shell by the wind tunnel experiment. This is smaller than 0.8 by the free falling experiment. The difference may be attributable to difference between the rotation and stationary state of the shell, or the experimental errors. At the moment, we cannot decide which is correct.

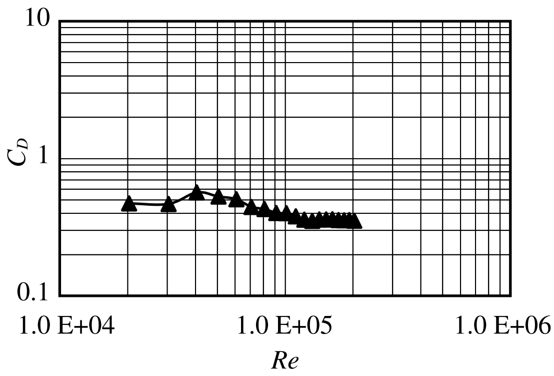

Fig. 4 C_D of a No. 5 sphere shell by wind tunnel.

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球形煙火玉の空気抵抗

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煙火玉の外部弾道に対する空気抵抗の効果を検討するために，球形模擬玉の自由落下実験と風洞実験を行った。自由落下の初期段階では，空気抵抗は無視でき，弾道計算の結果は異なる抵抗係数を用いても実質的には影響を受けないことがわかった。空玉の抵抗係数を自由落下及び風洞実験からあるレイノールズ数の領域で推定した。

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