Article

Analogy between wedge-induced steady oblique detonation and one-dimensional piston-supported unsteady detonation

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Abstract

Analogy between wedge-induced steady oblique detonation and one-dimensional unsteady piston-supported detonation is investigated based on a series of simulation results. The simulations are carried out by one and two-dimensional Euler Equations. Four types of wave structure of the wedge-induced oblique detonation appear in the simulation results. Those structures are basically the same as those of the piston-supported detonation, except for the type having the triple point on the oblique detonation wave. The reactant mass fraction history on the piston surface agrees well with that on the wedge wall in all types. A series of simulations of the one-dimensional piston-supported detonations varying activation energy, heat release, and piston speed are carried out in an attempt to understand a dominant parameter determining of the wave structure. We focus on reaction intensity as the dominant parameter. The reaction intensity, which is newly proposed as the ratio of the induction to total reaction time, represents the characteristic of the wave structure of the detonation. The reactant mass fraction history on the piston surface of the piston-supported unsteady detonation gives the induction and total reaction time. The reaction intensity of the piston-supported detonation classifies the wave structures in one-dimensional piston-supported detonation, and agrees well with that of the wedge-induced detonation.

1. Introduction

Aerospace propulsion systems utilizing the characteristics of detonation waves are being watched with interest as the next generation systems [1)~4)]. The fundamental physics on the detonation has not been fully elucidated to design an optimized engine of the aerospace propulsion systems. Suppose that the detonation wave holder is inserted in the supersonic combustible flow, the characteristics of the ignition and stabilization of detonation wave are totally different from the self-sustained detonation waves usually observed in the experiments. One of the concepts on the detonation-based aerospace propulsion system is the utilization of the stabilized oblique detonation wave [1)~4)]. The combustion behind the leading shock wave could be sustained by the holders such as the wedge, and the oblique detonation wave is settled at the supersonic combustor. Kasahara *et al.* [5)~8)] have studied the stabilities of oblique detonation around the wedge and spherical body using a two-stage light-gas gun. They reveal the wave structure and the critical condition of ignition. Many researchers have studied the attached wedge-induced detonation experimentally [9) 10)] and numerically [11)~15)]. Silva and Deshaies [13)] have simulated the wedge-induced detonation using the detailed chemical reaction model. Their goal is to determine the conditions leading to the stabilization of oblique detonation waves by a sharp wedge. They have also focused on the distance from the leading edge to the onset of the reaction zone. Papalexandris [14)] have investigated the effect of the wedge's top corner for the reaction zone behind the oblique shock.

The elucidation of the onset of the reaction zone on the wedge wall must be necessary for the development of the detonation-based aerospace propulsion system. In fact, the onset is strongly related with the wave structure and the stabilization of the wedge-induced oblique detonation [13) 14)]. In the present study, the two-dimensional wave structures of oblique detonation are tried to understand by the one-dimensional piston-supported detonation wave structures, analogically. All consideration on the analogy is numerically carried out. The analogy between the two-dimensional steady solutions and the one-dimensional unsteady solutions has been known since 1950's [16]. Ghorbanian and Sterling [15] have simply reported the analogy between the oblique detonation with double ramp and piston-supported detonation by a twice-accelerated piston. However, their report did not mention the physical meaning of the analogy. Accomplishment of the analogy must be the great help for the understanding of the fundamental physics on the wedge-induced oblique detonation, and makes easier to predict the two-dimensional physics.

2. Computational setup

The computational setup in the present study is the same procedure as that used in a previous study [17]. For the simulations of wedge-induced detonations, the grid is clustered to a triple point and a detonation wave front because of their chemical stiffness. We prepare a grid resolution of at least 10 points in the half-reaction length, $L_{1/2}$. For the simulations of piston-supported detonations, a grid resolution of 100 points/$L_{1/2}$ is prepared in the whole grid. The finer grid is also used to confirm the reliability of the resolutions for current works.

Table 1 Numerical condition:
U ; main flow speed, θ ; angle of wedge,
E ; activation energy, Q ; heat release parameter,
f_n ; degree of normal overdrive,
f_p ; degree of overdrive of piston.

Case	U	θ (°)	E	Q	f_n	f_p
1	9.255	27	10	50	1.200	1.111
2	20.580	20	50	50	2.000	1.865
3	11.509	35	50	50	1.830	1.722
4	12.171	20	40	50	1.132	1.103

Table 1 denotes the parameter of main flow conditions of wedge-induced oblique detonation, such as the main flow speed U, the angle of the wedge θ, the activation energy E, and the heat release parameter Q. U, E, and Q are non-dimensional values and normalized by $\sqrt{RT_0}$, RT_0 , and RT_0 respectively, where R and T_0 is the gas constant and the initial temperature. These conditions conform to those of the previous work [14]. The frozen shock strength initiated by the piston is set to be the same as the frozen oblique shock strength by the wedge, because the growth of the

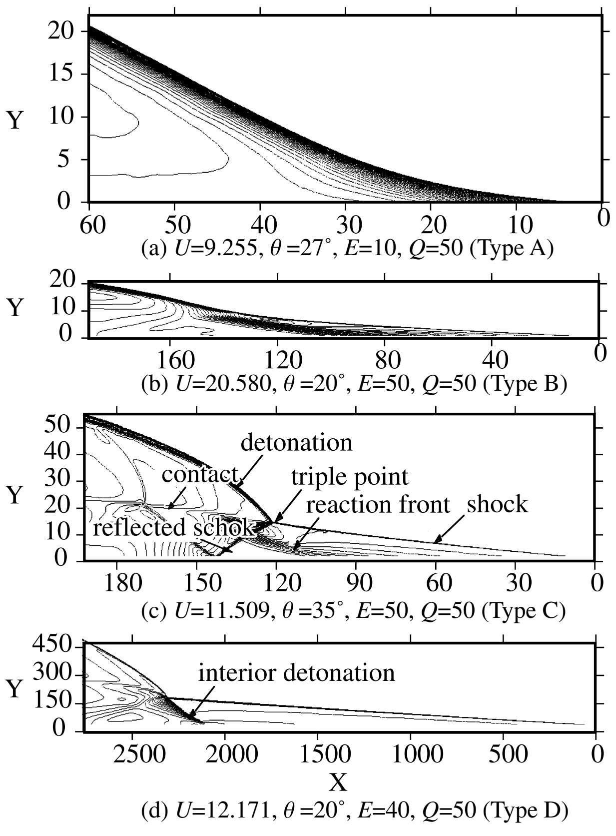

Fig. 1 The density distributions of the wedge-induced detonations. U ; incoming flow speed, θ ; wedge angle, E ; activation energy, Q ; heat release.

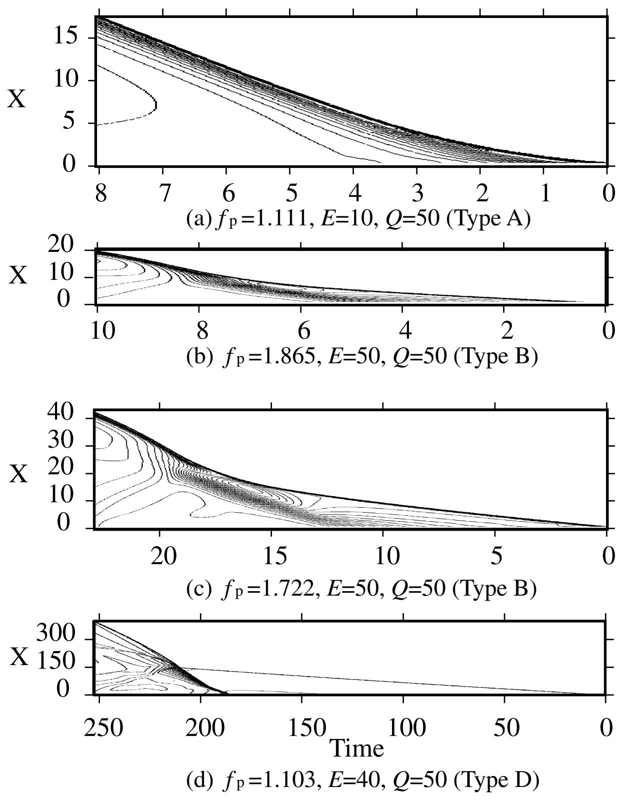

Fig. 2 T-x diagrams of density distributions of the one-dimensional piston-supported detonations. f_p ; degree of overdrive, E ; activation energy, Q ; heat release.

detonation wave front at the early stage is focused for the analogy. The degree of normal overdrive f_n, which is calculated by U, θ, E, and Q, is shown in the same table. Additionally, the degree of overdrive for the piston-supported detonation is indicated by f_p. The degree of overdrive is defined by $f = D^2/D^2_{CJ}$, is D detonation speed.

3. Result and discussion

In a wedge-induced oblique detonation, a tip of the wedge initiates an oblique shock, and a reaction occurs behind the oblique shock. Subsequently, the reaction front interacts with the oblique shock and finally develops into the oblique detonation. On the other hand, a piston, which moves at supersonic speed, generates a shock wave, and a reaction occurs behind the shock wave in a piston-supported detonation. The reaction wave soon catches up with the shock wave, and these waves develop into the detonation. As mentioned above, the basic wave components of a steady wedge-induced detonation, such as a leading shock, a reaction front, and a detonation, are the same as those of an unsteady piston-supported detonation. A series of simulation results of the wedge-induced oblique detonation demonstrates four types (Type A–D) of wave structure in Fig. 1, and are reliable because the features of these detonations are the same as those of the previous works [14]. Figure 2 shows the t–x diagrams of the density distributions of the one-dimensional piston-supported detonation, and the simulation conditions correspond to those in Fig. 1, as shown in Table 1. The x in Fig. 2 is the distance from the piston surface. The wave structure of each case in Fig. 1 is the same as the time-evolving distribution in Fig. 2, except for Figs. 1c and 2c. In Type A (Figs. 1a and 2a), the shock wave always couples with the reaction front and smoothly develops into the oblique detonation. The wave structure in Type B (Figs. 1b, 2b, and 2c) are slightly different from that in type A, because the shock wave is decoupled with the reaction front at the early stage of the shock initiation. In Type D (Figs. 1d and 2d), the shock waves are generated from the tip of the wedge and the piston surface. After a while, the reaction starts at the wedge and the piston surface, and expands gradually as an interior detonation. Eventually, the interior detonation penetrates the shock wave and becomes the detonation. Type C in Fig. 1c must be the unique feature in the two-dimensional flow, because the triple point is observed on the oblique shock. The oblique shock and the oblique detonation correspond to the incident shock and the mach stem, respectively. On the way to get the converged solution, the triple point and the reflected shock moves upstream, and finally settles down around x =120. This feature must be affected by the multi-dimensional effect and never happen in the one-dimensional flows.

In order to reveal the analogy between wedge-induced oblique detonation and one-dimensional piston-supported detonation, the history of reactant mass fraction Z, the pressure on the wall P_w, and the shock pressure P_s of each case are shown in Fig. 3. The time in the wedge-induced oblique detonation corresponds to the elapsed time of the flow along the wedge surface. In each case, the reactant mass fraction histories are almost the same between the wedge-induced oblique detonation and the one-dimensional piston-supported detonation. Therefore, the reactant mass fraction histories on the wedge surface can be predicted by the results of the piston-supported detonation. Types A and B have been recognized as the same smooth structure in previous work [13] but not in the work because the separation of shock wave and the reaction front exists at the early stage in Type B. Furthermore, the reactant mass fraction of Type B does not decreases as smoothly as that of Type A. However, the pressure histories of the wedge-induced detonation qualitatively agree with those of the piston-supported detonation. Both of Type C and D are the abrupt structures having the connected point of the oblique shock wave and detonation. See the specific feature of pressure history of Type C (wedge) in Fig. 3c, the peak of the shock pressure corresponds to the triple point,

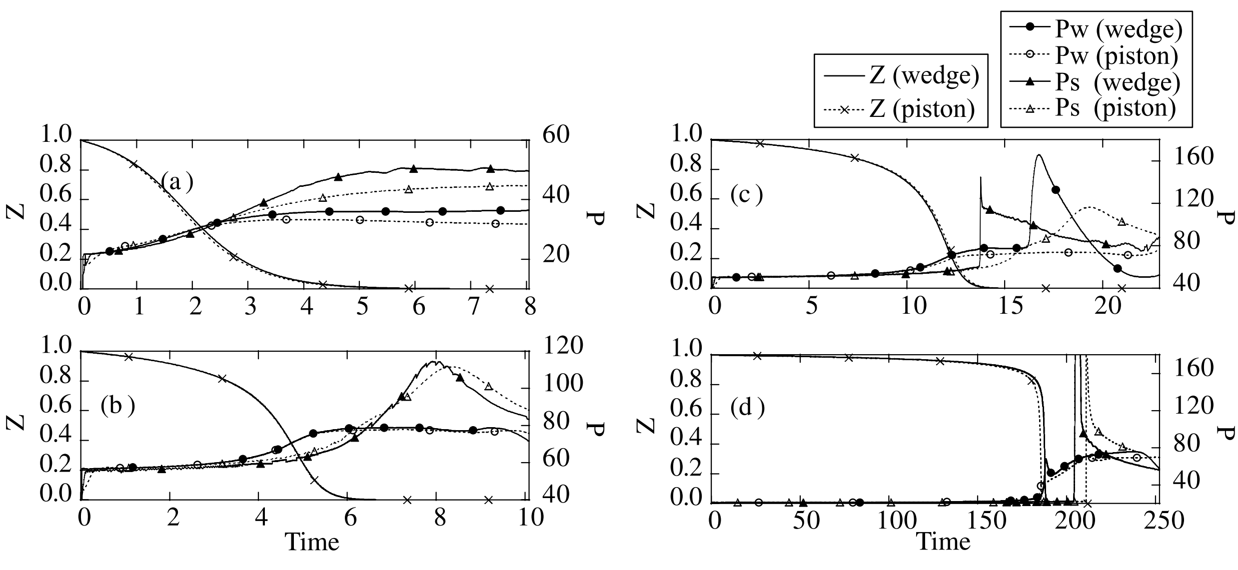

Fig. 3 The histories of the reactant mass fraction Z, the pressure on the wall P_w, and the shock pressure P_s.

and the peak of wall pressure does to the incidence of the reflection shock from the triple point. In Fig. 3c, the pressure histories of wedge-induced detonation are quantitatively different from those of piston-supported detonation. On the other hand, in Case 4, the shock pressure histories of Type D are quantitatively agree between the wedge-induced detonation and the piston-supported detonation in Fig. 3d. The abrupt structure is caused by the penetration of the interior detonation, which is observed in the both case of the wedge-induced and the piston-supported detonation. The sudden peak of shock pressure histories corresponds to the penetration of the interior detonation. The reactant mass fraction of Type D has a long induction time and decrease rapidly after the induction time. As mentioned above, each type has the specific reaction progress. The reaction progress that is affected by the shock strength dominates the wave structures of Type A, B, and D.

The detailed investigation on the simulation results clarifies that the wave structure type is dominated by the reaction progress, which is represented by the profile of the reactant mass fraction on the wedge wall and the piston surface. The characteristic reaction time is considered in order to quantitatively indicate the variation of the reactant mass fraction behind the leading shock wave. Figure 4 shows the reactant mass fraction history on the piston wall of the one-dimensional piston supported unsteady detonation. See Fig. 4 with the following ideas, the time of intersection between the tangent line whose gradient is maximum on the reactant mass fraction history and the line of Z=1 is defined as induction time t_i. The time of intersection between the tangent line and the line of Z=0 is defined as total reaction time t_r. The non-dimensional value is defined by t_i/t_r, which represents the reaction intensity. Table 2 shows the reaction intensity t_i/t_r on the wedge wall and the piston surface in all cases corresponding to the condition in Figs. 1 and 2. In each case in Fig. 3, the profiles of the reactant mass fraction on the piston surface are quantitatively the same as that on the wedge wall. A series of simulations of the one-dimensional piston-supported detonations varying activation energy, heat release, and degree of overdrive were carried out in an attempt to understand a dominant parameter determining the wave structure. Figure 5 shows the relation between the degree of overdrive and t_i/t_r, and the plots correspond the type of the wave structure of piston-supported detonations. The results clarify that the reaction intensity ti/tr dominates the wave structure in the one-dimensional piston-supported detonation and must be available for prediction of the

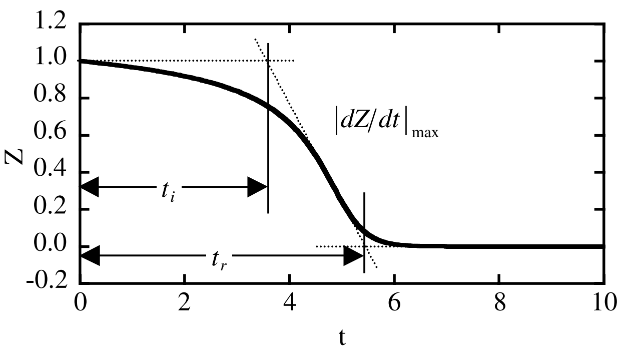

Fig. 4 The definition of the reaction intensity.

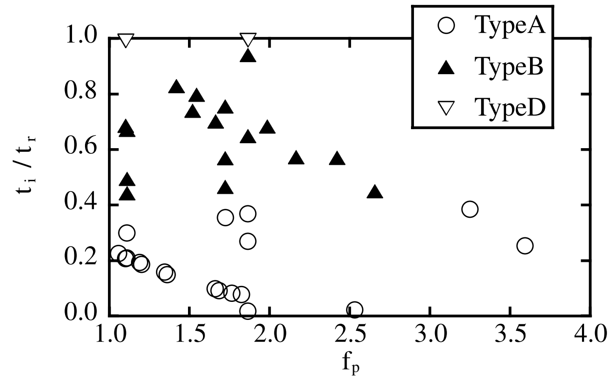

Fig. 5 The degree of overdrive versus the reaction intensity for piston-supported detonation.

Table 2 The reaction intensity.

Case	1	2	3	4
Wedge	0.212	0.648	0.758	0.993
Piston	0.210	0.649	0.756	0.992

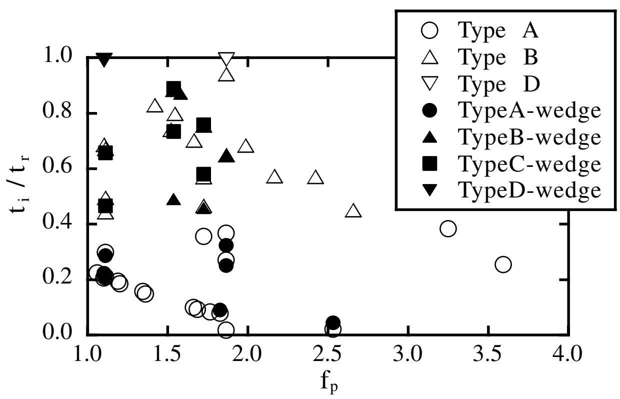

Fig. 6 The degree of overdrive versus the reaction intensity for wedge-induced detonation and piston-supported detonation.

wave structure observed in the wedge-induced oblique detonation as follows; t_i/t_r <0.4 (Type A), 0.4< t_i/t_r <0.99 (Type B or C), 0.99< t_i/t_r (Type D). Figure 6 shows t_i/t_r of the wedge-induced oblique detonation superimposed with that of the piston supported detonation. This comparison says that the t_i/t_r of the wedge-induced detonation agrees with that of the piston-supported detonation, and can be predicted by the t_i/t_r of the piston-supported detonation without consumption of CPU power. Next, we focus on the wave structure. The wave structure of the wedge-induced detonation is the same as that of the piston-supported detonation except for Type C, as shown in Fig. 1 and 2. Type B always appears when the simulation of piston-supported detonation carried out under the condition for Type C of the wedge-induced detonation. Therefore, The condition for Type B in one-dimension is supposed to be the necessary condition for the occurrence of Type C in two-dimension. In other words, the results of the piston-supported detonation indicate the potential for the wave structure with the triple point in the wedge-induced oblique detonation. The occurrence of Type C is dominated by a parameter other than t_i/t_r. The dominant parameter for determining the classification between Type B and C may be the wedge angle because the main flow speed and the wedge angle, that are the parameters of wedge-induce detonation, can be replaced with the piston speed in this analogy. The analogy between the two-dimensional steady solutions and the one-dimensional unsteady solutions has been known since 1950's. The analogy established in the previous work [16] when the shock angle was small. In the present study, the feature is also confirmed, but the reaction intensities on the wedge and piston wall accord well in any angle. This is useful for us to know the position of the reaction zone with a low cost of CPU power and to determine the wedge length.

4. Summary

Analogy between the wedge-induced steady oblique detonation and the one-dimensional piston-supported unsteady detonation was investigated based on the present numerical results. The four types of the wave structures of the wedge-induced oblique detonation appeared and were basically the same as those of the piston-supported detonation, except for the wave structure having the triple point on the oblique detonation wave. The reactant mass fraction histories on the wedge wall were almost identical with those on the piston surface for each case. A series of simulations of the one-dimensional piston-supported detonations varying activation energy, heat release, and piston speed were carried out in an attempt to understand the dominant parameter determining of the wave structure. The reaction intensity, which was proposed as the ratio of the induction to total reaction time using the reactant mass fraction histories on the piston surface, dominated the wave structure in the one-dimensional piston-supported detonation. The reactant mass fraction history of the wedge-induced detonation can be predicted by the results of the piston-supported detonation without time-consuming calculation. This prediction is useful for us to determine the wedge length that is the important factor to design the propulsion system such as the oblique detonation engine.

Acknowledgements

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