Determination of JWL equation of state parameters by hydrodynamically analytical method and cylinder expansion test

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JWL equation of state parameters for nitromethane (density of 1136 kg m^{-3}) and two TNT explosives (densities of 1563 kg m^{-3} and 1602 kg m^{-3}) were determined by means of the combination of hydrodynamically analytical method and cylinder expansion test. Cylinder expansion test not only laid much foundation for the analytical procedures to determine the JWL parameters of explosives but also provided a calibration standard for numerical calculations using JWL EOS. Through hydrodynamic and elastic-plastic-dynamic analysis on cylinder expansion movement, the isentropic pressures and specific volumes of detonation products might be related. The obtained pressure and specific volume data were then fitted by the isentropic form of JWL EOS to have the unknown parameters in JWL EOS be determined. Due to the fact that cylinder expansion test was unable to provide any information of detonation products at large volumes, we investigated several possible ways to take such situation into account and several sets of JWL parameters for each explosive were obtained individually. The obtained JWL EOSs were employed in the numerical calculations to reproduce cylinder expansion tests. Finally, a best set of JWL parameters for an explosive was found through comparisons of calculations and experiment.

1. Introduction

Explosives are commonly applied for doing work toward the surrounding media via the expansion of their detonation products. In order to assess such kind of performance of an explosive, it is necessary to know the equation of state (EOS) of its detonation products. Up to now, a number of EOS formulations have been available for describing the behavior of detonation products. Of those, the Jones-Wilkins-Lee (JWL) EOS may be the most commonly used one for workers in this field due to both its simple form in the expression and its establishment on the experimental basis. The JWL EOS has the form of [1]

$$P = A\left(1-\frac{\omega}{R_1 V}\right)e^{-R_1 V} + B\left(1-\frac{\omega}{R_2 V}\right)e^{-R_2 V} + \frac{\omega E}{V}, \qquad (1)$$

and its adiabat (isentrope) is given by

$$P_s = Ae^{-R_1 V} + Be^{-R_2 V} + \frac{C}{V^{\omega+1}}, \qquad (2)$$

where, P is the pressure, E is the internal energy per unit volume, and V is the relative volume equivalent to the ratio of the current specific volume over the initial specific volume of the explosive, P_s is the pressure of detonation products on the isentropic adiabat, A, B, C, R_1, R_2, and ω are parameters required to be determined. An

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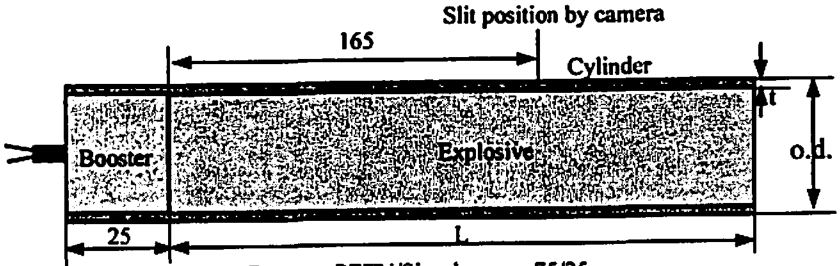
experimental system called cylinder expansion test is used for the calibration of the JWL equation. Cylinder expansion test gives the wall motion history via a high speed camera for recording the expansion of the cylinder wall by the pull of the detonation products from the detonation of the explosive filled in the cylinder. Two approaches have been available for the determination of JWL parameters. One and the earlier way is a trial and error procedure that uses the hydrodynamic code and some theoretical constraints to find the best fit to the wall motion history [1]. Another is an analytic method that converts the wall motion history into a pressure and volume data for the detonation products of the studied explosive through the analytical manipulation on cylinder expansion test experiment according to hydrodynamic and elastic-plastic dynamic theories, and then, uses Eq. (2) to best fit the acquired pressure and volume data.[2][3] The advantage of later approach is that it can avoid the complicated numerical computation so that it can be easily mastered. This paper presents the studies on the acquirement of JWL EOSs for explosives of 1136 kg m^{-3} nitromethane (NM), 1563 kg m^{-3} TNT and 1602 kg m^{-3} TNT following the later approach. Different from the original formula, we particularly explore the influences of the large expansion domain, which exceeds the range of the general cylinder expansion data, to the determination of JWL parameters. For each explosive, several JWL EOSs are obtained. Those JWL EOSs are, instead, employed in a hydrodynamic code to examine which one is best able to describe the motion history of the cylinder wall.

2. Experimental

2.1 Cylinder expansion test

Fig. 1 schematically illustrates the cylinder expansion test system in an experiment. Fig. 1a shows the cylinder assembly used for the experiment and Fig. 1b gives the recoding system for the expansion process of the cylinder wall by



Booster: PETN/Si-polymer = 75/25

Fig.1 Configuration of cylinder and explosive system for cylinder expansion test.

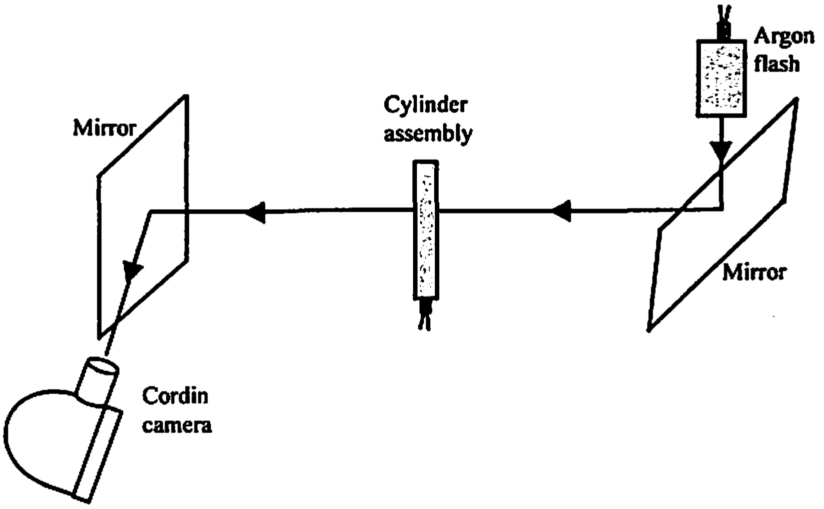
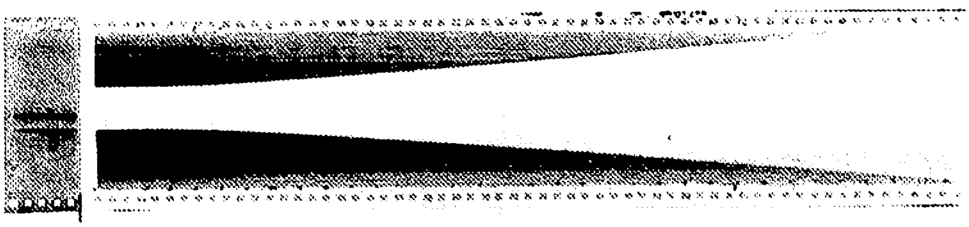


Fig.2 Illustration of streak photography for measuring the cylinder expansion.



Static image Dynamic image 30 mm | 3 microseconds

Fig.3 A typical streak camera picture obtained from cylinder expansion test.

means of high speed camera. The cylinder used in the experiments was an OFHC copper tube that underwent the heat treatment under vacuum condition at 600 degrees centigrade for one hour to make the residual stresses be eliminated. For each explosive test, the cylinder was of 30 mm in outer diameter and 300 mm in length, however, the wall was slightly varied due to the different phases for experimental preparation. The variation of the thickness of cylinder wall, respectively, is 2.5 mm for nitromethane, 2.26 mm for 1563 kg m^{-3} TNT and 2.28 mm for 1602 kg m^{-3} TNT. The wall of the cylinder was precisely machined to reach a precision within the error less than 2 percentage. The explosive was filled into the cylinder at room temperature and a mixture of pentaerythritol tetranitrate (PETN) and silicon rubber in a weight ratio of 75 over 25 was used as the booster for the initiation of the explosive in the cylinder. The booster was fired by an electric bridge wire (EBW) detonator. A rotating streak camera (Cordin 116, USA) using shadowgraph technique was employed to measure the expanding process of the cylinder at sweeping speed of 12000 m s^{-1}. The viewing slit of 0.1 mm width was aligned at a position 165 mm away from the end of the cylinder that was detonated early. After each shot, the film was read with an optical microscope of a space resolution of 0.001mm to provide the expansion history of the cylinder. Fig. 3 presents a typical streak picture obtained from such experiments in which the nitromethane was used as the explosive. It shows clearly that the portion of the cylinder locating at the viewed cross-section spreads outside with the elapsing of the time. Because the picture is obtained by directly scanning from the negative film, the portion occupied by the cylinder, then, exhibits a bright area and the area beyond the cylinder has a dark region.

2.2 Detonation velocity measurement

Detonation velocities for nitromethane and two TNT were measured by an electrically charged pin technique independently of the cylinder expansion test experiments. Eight pairs of pins were settled at the beginning of 120 to 260 mm away from the booster end with an interval of 20 mm along the cylinder axis. The accuracy is within 0.5% for the yielded detonation velocity. The average detonation velocity for nitromethane is 6271 m s^{-1}, 6581 km s^{-1} for TNT 1560 kg m^{-3}, and 6918 m s^{-1} for 1602 kg m^{-3}.

3. Isentropic pressures and specific volumes of detonation products

3.1 Mathematical expression on expansion history

In order to establish an analytical procedure to gain JWL EOS parameters, the first thing should be done is to give a mathematical description the expansion history. It may be accomplished by the non-linear function fitting method using least-square fit skill 1). From the experiment, a lot of data on displacement and time for the cylinder expansion could be acquired, but they are in a discretization form and should be continuously related with a smooth function. Considering that the pressure in detonation products can be approximated by an exponential form to express its decaying trend, thus, the acceleration of the cylinder is also able to be expressed by an exponential function of time, for instance, by

$$\frac{d^2r}{dt^2} = \sum_{j=1,n} a_j b_j^2 e^{-b_j(t-t_0)}, \qquad (3)$$

where, r denotes the radial coordinate of the cylinder, t is the time, t_0 is a delay in time, and a_j and b_j are fitting constants. Integrating Eq. (3) may result in the acquirement of the velocity and displacement of the cylinder varying with time as follows

$$\frac{dr}{dt}=\sum_{j=1,n}a_j b_j[1-e^{-b_j(t-t_0)}], \quad \text{(for velocity)} \qquad (4)$$

$$r-r_0=\sum_{j=1,n}a_j[b_j(t-t_0)-(1-e^{-b_j(t-t_0)})], \quad \text{(for displacement)} \qquad (5)$$

where r_0 is the initially radial coordinate of the cylinder. Making use of the experimental data and fitting them with Eq. (5) leads to the solution of the constants of a_j and b_j with the aid of following constraints: (1) the initial displacement is zero; (2) the initial velocity is zero; (3) the final expansion velocity is constant; and (4) the initial pressure acting on the cylinder is the C-J pressure of the explosive. In practice, only two terms (n=2) is adequately enough to make a well fit to the experimental data.

3.2 Pressures of detonation products

In the above section, the obtained relationship on displacement and time refers to that of the cylinder in a fixed frame. However, in the application of hydrodynamic and elastic-plastic dynamic theories, it becomes necessary to follow the motion of the particle of the cylinder for the analytical formulation. Before doing so, some assumptions should first be made for convenience. (1) the detonation wave is one-dimensional; (2) the detonation products are inviscid and immediately transferred from the explosive; (3) the cylinder wall is incompressible; (4) the reverberation of waves in cylinder wall is ignored. With the expansion of the cylinder, not only does the exterior radius, r, change, the radius of mass center (the position that equally divides the mass in the cross-section of the cylinder), r_m, and the interior radius, r_i, of the cylinder, also changes. From assumption (3), those radii have the following relations,

$$r_m=\sqrt{r^2-\frac{1}{2}(r_0^2-r_{i0}^2)}\ , \qquad (6)$$

$$r_i=\sqrt{r^2-(r_0^2-r_{i0}^2)}\ , \qquad (7)$$

where, the subscript 0 denotes the values of quantities before the expansion. Because in the later analyses to solve the pressures and specific volumes of detonation products, the mass center radius, r_m, is the main variable involved, it will become more convenient to make the curve fitting to the mass center radius, r_m, not to the exterior radius, r, directly.

To an infinitesimal element particle of the cylinder, the pressure acting on it may be expressed by

$$P=\frac{M}{2\pi r_i}\left[1+\frac{1}{D^2}\left(\frac{dr_m}{dt}\right)^2\right]^{-\frac{3}{2}}\frac{d^2r_m}{dt^2}+\sigma\frac{(r^2-r_i^2)}{(2r_m r_i\ )}\ , \qquad (8)$$

where, P is the pressure of detonation products, D is the detonation velocity of the explosive, M is the mass of the cylinder per unit length, and σ is the hoop stress in the cylinder, which is equal to E $(r_m/r_{m0}$-1$)$, E is the Young's modulus. When $\sigma>\sigma_r$, then $\sigma=\sigma_r$, where σ_r is the yield stress. To the OFHC, E is chosen to be 103.35 GPa and σ_r to be 0.31 GPa. The details on the formulation may be found in Ref. 3.

3.3 Specific volume of detonation products

The specific volume of the detonation products may be obtained through the ideal detonation wave theory and the isentropic flow of the detonation products in a varying cross-sectional tube. The result is

$$V=\frac{1}{\rho_0}\left(\frac{r_i}{r_{i0}}\right)^2\left[1-\frac{P}{\rho_0 D^2}\left(\frac{r_i}{r_{i0}}\right)^2+\frac{M}{C}\left[1-\sqrt{1+\frac{1}{D^2}\left(\frac{dr_m}{dt}\right)^2}\right]\right]\ . \qquad (9)$$

where, C is the mass of the explosive per unit length and ρ_0 is the initial density of the explosive. The pressures and specific volumes of detonation products are related through an intermediate quantity, r, of the cylinder displacement. The calculated pressures and specific volumes are taken to be those along the isentropic adiabat of detonation products.

4. Determination of JWL parameters

It is generally acceptable that the detonation products start the isentropic release from the Chapman-Jouguet (C-J) state in an ideal detonation model. Due to the fact that the C-J state cannot be correctly implied from the cylinder expansion test, the state variables of the detonation products at the C-J state should be measured or calculated by other ways. At the C-J state, however, three conditions for detonation products: (1) C-J pressure holds at the C-J specific volume; (2) isentropic adiabat is tangent to the Rayleigh line; (3) The internal specific energy is equal to the addition of the work done during the compression and the detonation heat of the explosive; should be satisfied. Thus, when using Eq. (2) to represent the equation of state of detonation products, the following equations are obtained,

$$P_{CJ} = Ae^{-R_1V_{CJr}} + Be^{-R_2V_{CJr}} + \frac{C}{V_{CJr}^{\omega+1}}, \qquad (10)$$

$$\rho_0 D^2 = A R_1 e^{-R_1V_{CJr}} + B R_2 e^{-R_2V_{CJr}} + \frac{C(\omega+1)}{V_{CJr}^{\omega+2}}, \qquad (11)$$

$$E_0 + \frac{1}{2}P_{CJ}(1.0 - V_{CJr}) = \frac{A}{R_1}e^{R_1V_{CJr}} + \frac{B}{R_2}e^{R_2V_{CJr}} + \frac{C}{\omega V_{CJr}^{\omega}}, \qquad (12)$$

where, P_{CJ} is the detonation pressure and, V_{CJr} is the relative volume of the detonation products (specific volume, V_{CJ}, to initial specific volume, V_0.) at the C-J state, ρ_0 is the initial density of the explosive ($\rho_0 = 1/V_0$), E_0 is the detonation heat of the explosive per unit volume. Except that the detonation velocities are measured from the experiments, the C-J detonation pressures and specific volumes and the detonation heats of the studied explosives are calculated from the CHEETAH V2.0.[5] For nitromethane and two TNT's, those values are listed in Table 1. Usually, R_1, R_2 and ω are chosen as the independent variables in the fitting procedure. If so, A, B, C can easily be solved from the above equations.

On the other hand, the cylinder expansion test is also not able to provide the pressure and specific volume data at the states with large specific volumes due to the experimental limitations of the relatively short photographic time period or the possible rupture of the cylinder at the later phase of photography. A general treatment is to assume that the detonation products at large specific volumes are identical to the ideally perfect gas. According to Eq. (2), the last term at the right hand side should play a dominant role for the contribution to the pressure (This is what was pre-considered by JWL during the establishment of JWL EOS). Therefore, parameter ω is limited within variation between 0.2 and 0.4 during the course of parameters' determination. However, in this study, we choose another four possible ways to investigate the affect of parameter ω to the accuracies of JWL parameters:

(1) Three sets of pressure and specific volume data at large volumes (V = 40, 80, and 160) from CHEETAH V2.0 calculations are added to those determined from the above cylinder expansion test and these data are fitted by Eq. (2). ω as well as other JWL parameters are determined by direct fitting with least-square fit method.

(2) The pressure and specific volume data obtained from CHEETAH V2.0 calculations are used as the fitting data without use of any data from the cylinder expansion test and the fitting method is performed to give parameter ω. This ω is kept unvaried and used to fit the data from the cylinder expansion test so that other JWL parameters are determined.

(3) Assuming that the detonation products are composed of matters of carbon, water, nitrogen, and carbon monoxide. Then, for mixtures of such species, the isentropic exponent, γ, is estimated by taking into consideration of the mole numbers in the chemical decomposition of explosives. Parameter ω is determined from $\omega = \gamma$-1. For instance, nitromethane and TNT,there are chemical decompositions of $CH_3NO_2 = 1.5H_2O + 0.5CO + 0.5N_2 + 0.5C$ and $C_7H_5N_3O_6 = 2.5H_2O + 3.5CO + 1.5N_2 + 3.5C$. For the diatomic ideal gases of nitrogen and carbon monoxide, γ equals to 7/5, and for the polytropic gas of water vapor, γ equals to 4/3. As a result, the estimated value of γ for

Table 1. The main detonation properties of NM and TNT.

	ρ0 (kg m^{-3})	D (m s^{-1})	PCJ (GPa)	E$_0$ (GPa m^3 m^{-3})
NM	1136	6271	12.50	5.750
TNT(1)	1563	6581	16.83	7.133
TNT(2)	1602	6918	17.86	7.381

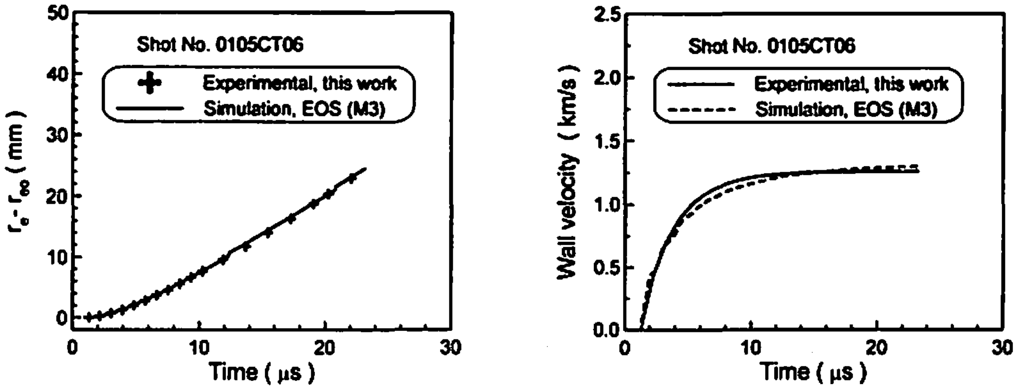
nitromethane is 1.360 and 1.377 for TNT. Hence, their parameters of ω. are 0.36 and 0.377, respectively. After the acquirement of parameter ω, the remaining work is to fit the cylinder expansion data using Eq. (2) to obtain the other JWL parameters.

(4) Use Eq. (2) to directly fit the cylinder expansion test data and parameter ω and other JWL parameters are determined from the fit.

Following the above principles, the JWL parameters of nitromethane and two TNT's are determined and are presented in Table 2. The obtained JWL EOSs are incorporated into a 2D Lagrangian code [6] for the numerical calculation tests. The calculated results show that the JWL parameters determined by method 3 are able to give the most accurate reproduction of the cylinder expansion test data for nitromethane, however, for two TNT's, method 1 is the best choice. Therefore, the final JWL parameters for nitromethane and two TNT's may be chosen and are presented in Table 3. The calculated cylinder expansion test using those JWL parameters for nitromethane and two TNT's are depicted in Figs. 4 through 6 together with the experimental data for comparison.

5. Conclusions

JWL equation of state parameters for nitromethane and two TNT explosives were



(a). (b)

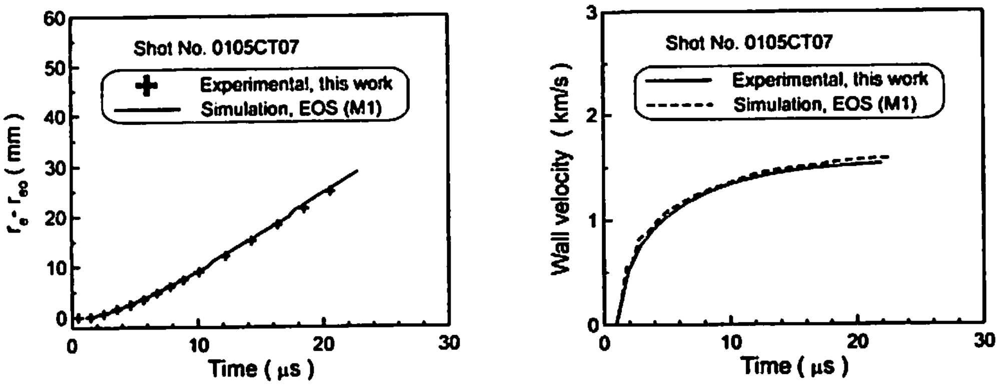
Fig.4 Comparison of experiment and calculation on cylinder expansion test for explosive of nitromethane. (a) displacement; (b) wall velocity.

Table 2. The obtained JWL parameters for NM and TNT from four methods.

Expl.	Method	A(GPa)	B(GPa)	C(GPa)	R1	R2	ω
NM	1	568.301	63.361	1.934	7.846	3.004	0.469
	2	369.861	37.749	1.418	6.377	2.455	0.375
	3	363.536	35.983	1.342	6.290	2.398	0.360
	4	813.032	76.011	2.153	8.822	3.222	0.508
TNT(1)	1	452.70	1.027	0.848	4.50	0.51	0.20
	2	450.302	1.602	1.413	4.542	1.756	0.264
	3	458.313	0.597	1.268	4.552	0.335	0.377
	4	464.397	0.223	1.852	4.622	0.488	0.350
TNT(2)	1	451.801	1.415	1.231	4.531	1.123	0.20
	2	478.371	2.017	1.317	4.608	0.783	0.290
	3	476.466	1.285	1.995	4.651	0.988	0.377
	4	470.857	1.086	1.691	4.598	0.640	0.35

Table 3. The best found JWL parameters of NM and two TNT's .

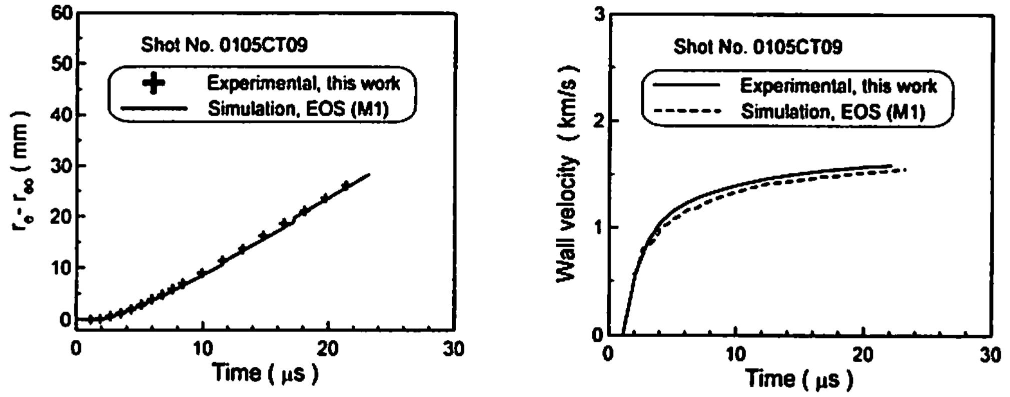
	A(GPa)	B(GPa)	C(GPa)	R1	R2	ω
NM	568.301	63.361	1.934	7.846	3.004	0.469
TNT(1)	452.70	1.027	0.848	4.50	0.51	0.20
TNT(2)	451.801	1.415	1.231	4.531	1.123	0.20



(a).

(b)

Fig.5 Comparison of experiment and calculation on cylinder expansion test for explosive of TNT with density of 1563 kg m^{-3}. (a) displacement; (b) wall velocity.



(a).

(b)

Fig.6 Comparison of experiment and calculation on cylinder expansion test for explosive of TNT with density of 1602 kg m^{-3}. (a) displacement; (b) wall velocity.

determined by means of the combination of hydrodynamically analytic method and cylinder expansion test. Due to the fact that cylinder expansion test was unable to provide any information of detonation products at large volumes, we investigated several possible ways to take such situation into account and several sets of JWL parameters for each explosive were obtained individually. The obtained JWL EOSs were employed in the numerical calculations to reproduce cylinder expansion tests so that a best set of JWL parameters for an explosive was found.

Acknowledgments

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