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THEORY OF THE BURNING RATE OF INDUSTRIAL SAFETY FUSE

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A semi-empirical formula which represents the relation among burning rate, pressure of combustion products and density of black powder core is presented.

Under the condition of stationary burning which is realized in routine test in the open atmosphere equations for the relations among the amount of lateral discharge, axial discharge of combustion products and the distance travelled by combustion front are derived.

The method for empirical determination of the coefficients involved in the equations are described.

An equation is derived for the total time of combustion for a safety fuse of a given length under the condition of laterally complete tamping which is nearly realized in practical use in boreholes. From this relation the exponent in the burning rate equation can be determined. Empirical results which confirm the equations are described with discussion on the stability of burning.

Introduction

While safety fuse has been widely used in industrial blasting for the ignition of black powper charge or detonation caps, no theoretical analysis on its combustion charactristics has been attempted. From the points of view of safety and efficiency in blasting, a safety fuse must be reliable in its uniformity and stability of rate of burning which are greatly influenced by conditions under which it is burnt.

Theory

I. Pressure and burning rate

From the practical point of view, the burning velocicty (v cm/sec) of a solid explosive has been expressed by previous authors, as a function of pressure (p kg/cm^2) at which it burns. Most of these functions have been empirical although some theoretical deductions have been attemped.

The most classical but convenient function is the following which may be called as "n-th power formula."

$$v_s=a_1p^n \qquad (1)$$

Then, the two-terms function which may be called a "linear formula" has been presented

$$v_s=a_2+b_2p \qquad (2)$$

From the standpoint of the kinetic theory of gas, the following function which may be called "proportional formula" has been suggested.

$$v_s=b_1p \qquad (3)$$

In the present author's a opinion[1] none of the three equations described above can represent the behaviour of pressure~rate curves over wide range of pressure and he suggested a more general function, that is,

$$v_s=a_3p^n+b_3p \qquad (4)$$

It is considered that (3) holds for high pressure range while (2) holds in medium pressure range excluding near zero pressure. For low pressure range including near zero pressure, the function (1) is the most rational and convenient at least at present.

II. Temperature of combustion and burning rate

In the author's simplified assumption

over low pressure range, the activation of solid explosive surface by the collision of combustion products plays a smaller part for combustion while the activation by the radiation from hot gases to cool surface plays a major roll. This energy transfer may be expressed to the first approximation by $p^n T^h$, where T is the temperature of the burnt gases. For the radiation from solid body (the combustion residue), $n=0$, $h=1$, while for gaseous CO_2[2], $n={}^1/_3$, $h=3.5$, for gaseous H_2O, $n={}^4/_5$, $h=3$ and other gases such as O_2, N_2, show no remarkable radiation.

For high pressure range, the activation of cool surface by the collision of active gaseous molecules plays a major part, therefore, the temperature of combustion T might affect the combustion rate by the relation $pe^{-E/RT}$ where E=activation energy and R=gas constant. There for in general,

$$v=a'T^h p^n+b' pe^{-\frac{E}{RT}}$$

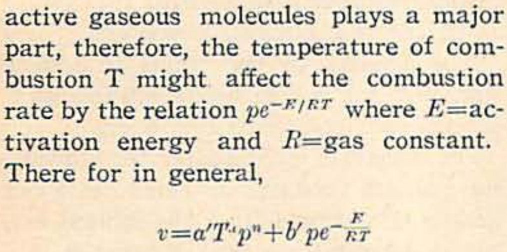


Fig. 1. Arrangement of powder grains along an axis of a Fuse.

Ⅲ. Loading density and burning rate

It may be assumed that a grain of powder under discussion is a solid cube whose side length is l_s, and that the distance between two grains along the direction of burning is l_g as is schematically shown in Fig. 1. In Fig. 1, v_s and v_g represent the linear burning velocities through the solid grain proper and through gaseous products along the direction of burning respectively.

It is assumed that the propagation of combustion from one grain to another is through gaseous medium. First, if the density of core Δ g/cm³ is assumed to be 1, and the number of layers of grains within 1 cm along the axis of a safety fuse, is expressed by M, then we have,

$$M(l_g+l_s)=1 \text{ cm}.$$

Time of combustion for 1 cm thickness is represented by t.

$$\frac{M(l_g+l_s)}{v}=M\frac{l_g}{v_g}+M\frac{l_s}{v_s}=t$$

If we transfer the density Δ_0 to $q^3\Delta_0$, then the total thickness of the solid becomes Mql_s, while that of gaseous space decreases to $Mq\times\frac{l_g}{q}=Ml_g$.

If we take $\Delta_0=1$, $\Delta=\Delta_0 q^3$, then

$$t=qM\frac{l_g}{q}\frac{1}{v_g}+qM\frac{l_s}{v_s}=Mq\left(\frac{l_g}{v_g}\frac{1}{q}+\frac{l_s}{v_s}\right)$$
$$=M\Delta^{\frac{1}{3}}\left(\frac{l_g}{v_g}\frac{1}{\Delta^{\frac{1}{3}}}+\frac{l_s}{v_s}\right)=\frac{Ml_g}{v_g}+\frac{Ml_s}{v_s}\Delta^{\frac{1}{3}}$$

or burning velocity

$$v \text{ cm/sec.}=\frac{1}{t}=\frac{v_g v_s}{M(l_g v_s+l_s v_g \Delta^{\frac{1}{3}})},$$

With the substitution $Mq\frac{l_g}{q}+Mql_s=1$ we obtain, $l_g=\frac{1}{M}-ql_s$

$$v=\frac{v_g\ v_s}{v_s+Ml_s(v_g-v_s)\Delta^{\frac{1}{3}}}$$

If the real density of the solid is indicated by S g/cm³, then $Sl_s^3M^3=\Delta_0=1$, therefore, $Ml_s=S^{-\frac{1}{3}}$

$$v=\frac{v_g v_s}{v_s+(v_g-v_s)\Delta^{\frac{1}{3}}S^{-\frac{1}{3}}} \qquad (5)$$

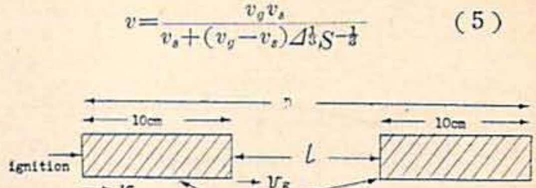


Fig. 2. Procedure of justification for $v_g \gg v_s$

In Fig. 2, two pieces of a safety fuse, each piece being 10 cm in length, are separated by a distance l along the direction of burning. When l=10 cm, the total time of combustion is about 25.4 sec. for a typical brand of safety fuse, while this fuse with a length of 20 cm burns within

25 sec. By these measurements, we may estimate that the velocity of propagation through space is at least $v_g=\frac{10}{0.4}=25$ cm/sec., with that for the fuse proper $v=\frac{20}{25}=0.8$ cm/sec., therefore, we may assume that

$$v_g \gg v_s \qquad (6)$$

Then we obtain

$$v=v_s\left(\frac{S}{\Delta}\right)^{\frac{1}{3}} \qquad (7)$$

R. Schwab[3] has derived a different function for the relation between loading density and detonation velocity of powdery explosives assuming for that case

$$v_g \ll v_s \qquad (8)$$

From the explanation above described, the general formula for burning velocity may be written as follows

$$v=\left(aT^{\lambda}p^{n}+be^{-\frac{E}{RT}}p\right)\left(\frac{S}{\Delta}\right)^{\frac{1}{3}} \qquad (9)$$

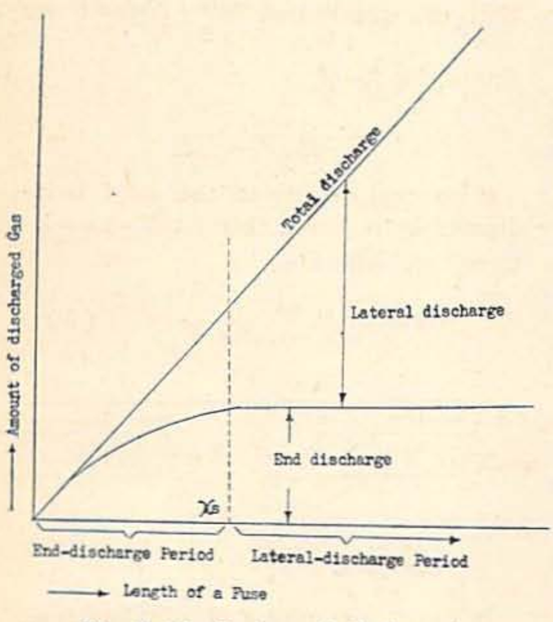


Fig. 3. Distribution of discharged gas from a safety fuse.

In the present paper which deals with a problem over a range of lower pressure, a simplified form of (9) may be used, that is,

$$v=aT^{\lambda}p^{n}\left(\frac{S}{\Delta}\right)^{\frac{1}{3}} \qquad (9)'$$

or

$$v=Ap^{n}\left(\frac{S}{\Delta}\right)^{\frac{1}{3}} \qquad (9)''$$

IV. Steady burning of a safety fuse

Experience has shown that an industrial safety fuse burns with uniform burning rate in open atmosphere which is the case in routine tests.

Exactly speaking, the relation between the total burning and the total length of a fuse L might be expressed by

$$t=A_1+B_1L$$

where A_1 and B_1 are constants.

As practically A_1 is very small, we may assume without serious error that $t=B_1\cdot L$, that is, the burning is steady.

The distance of the travel of a combustion front is measured from the ignited end and is indicated by x cm.

For a certain period after the ignition, the gaseous products of combustion are mainly discharged from the ignited end because lateral resistance for the discharge of gas is greater than that for axial discharge. This period may be described as "end-discharge period", while after this period the discharge is carried out through the lateral porous wall of a fuse and this second stage may be called "lateral discharge period".

The amount of discharge G_t per unit time is, according to the rules of thermodynamics, as follows.

$$G_t=\varphi\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}\sqrt{\frac{2gk}{k+1}}\frac{F}{\sqrt{RT}}\,p \qquad (11)$$

Where k = ratio of the specific heats of the gaseous products at constant pressure to that at constant volume, g= the constant of gravity, F = effective sectional area of discharge, R=gas constant, T= temperature of gas, p=pressure of gas, and φ is the coefficient of resistance for discharging flow. As the resistance for gaseous discharge along the axis of a fuse increases as the combustion proceeds, φ may be approximately expressed by the following function

$$\varphi=\frac{\varphi_0}{1+bx} \qquad (12)$$

where φ_0 is the value of φ at $x=0$.

If q denotes the effective area for lateral discharge through porous structure of the side wall of a fuse for unit length (1cm) of a fuse, then the total effective area available for the discharge of the gaseous products may be expressed by $\pi D^2/4+qx$, where D is the diameter of the section of a powder core.

Therefore at any distance x

$$\varphi F=\varphi_0(\pi D^2/4+qx)/(1+bx) \quad (13)$$

In steady state of burning the value of φF should be independent of the distance x, thus we obtain the following condition.

$$F_0=\frac{\pi D^2}{4}=\frac{q}{b} \quad (14)$$

If x is small

$$\varphi F=\varphi F=\varphi_0\left(\frac{\pi D^2}{4}\right)=\varphi_0 F_0 \quad (15)$$

where F_0=sectional area of the powder core.

If x is large $\quad \varphi F=\varphi_0\frac{q}{b} \quad (15)$

(15) and (15)′ mean that shortly after the ignition the discharge of gas is mainly realized through an ignited end while after a certain period the gas is discharged through a lateral wall.

(a) Amount of discharge in "end-discharge period."

If we define $K=\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}\sqrt{\frac{2gk}{k+1}}\frac{1}{\sqrt{RT}}$

(a powder constant) (16)

where $k=1.25$, $RT=f$, (force of powder) then we obtain

$$K=\frac{20.7}{\sqrt{f}} \quad \text{cm/sec.}$$

In steady burning the total amount of gaseous discharge per unit time is

$$G_t=(\varphi_0 F_0 K)p \quad (17)$$

Amount of discharge per unit time through the ignited end is

$$G_e=\varphi_0 F_0\frac{1}{1+bx}Kp \quad (18)$$

Amount of discharge per unit time through lateral wall is:

$$G_s=\varphi_0\frac{qx}{1+bx}Kp=\varphi_0\frac{F_0 bx}{1+bx}Kp \quad (19)$$

Of course the relation $G_t=G_e+G_s$ holds. For the total length of a fuse L cm the total amount of gas:

$$G_t L=(\varphi_0 F_0 Kp)L \quad (20)$$

the amount of discharge from ignited end.

$$G_e L=\int_0^L \varphi_0 F_0\frac{1}{1+bx}Kpdx=(\varphi_0 F_0 Kp)\cdot\left\{L-\frac{2.303}{b}\log(1+bL)\right\} \quad (21)$$

the amount of lateral discharge:

$$G_s L=\int_0^L \varphi_0 F_0\frac{bx}{1+bx}Kpdx=(\varphi_0 F_0 Kp)\frac{2.303}{b}\log(1+bL) \quad (22)$$

If we define $\frac{G_s L}{G_e L}=r$,

$$\text{then } r+1=\frac{G_t L}{G_e L}=\frac{0.42436L}{\log(1+bL)} \quad (23)$$

As the ratio r or $r+1$ are easily determined by simple experimentation, we can find the value of the coefficient b.

(b) Amount of discharge in a "lateral discharge period".

Total amount of gaseouns discharge is the same with that in case (a), that is,

$$G_t L=(\varphi_0 F_0 Kp)L \quad (20)$$

The amount of axial discharge remains the same after $x>x_s$ where x_s denotes the length which corresponds to the period of end discharge.

$$G_e L=(\varphi_0 F_0 Kp)\left\{x_s-\frac{2.303}{b}\cdot\log(1+bx_s)\right\}=\text{const} \quad (24)$$

The amount of lateral discharge after $x>x_s$ is as follows:

$$G_s L=(\varphi_0 F_0 Kp)L-(\varphi_0 F_0 Kp)\cdot\left\{x_s-\frac{2.303}{b}\log(1+bx_s)\right\}=(\varphi_0 F_0 Kp)L-\text{const.} \quad (25)$$

V. Velocity of burning in a stationary condition.

If we denote the density of powder core by Δ, the weight of powder per unit length of a fuse is $\pi/4D^2\Delta$. If a fuse burns with burning velocity v, the weight of powder burnt per unit time is $\pi/4\ D^2\Delta v$. In steady burning this weight is the same with the total amount of discharge per unit time, that is $G_t=\varphi_0 F_0 Kp$, if we assume there remains no solid residue. If there is some solid residue, this amount can be taken into calculation modifying the value of φ_0.

Thus we obtain $\varphi_0 Kp=\Delta v$.

With the relation $v=Ap^n\left(\frac{S}{\Delta}\right)^{\frac{1}{3}}$ (9)″, we find $\varphi_0 Kp=Ap^n S^{\frac{1}{3}}\Delta^{\frac{2}{3}}$, therefore,

$$p=\left(\frac{AS^{\frac{1}{3}}\Delta^{\frac{2}{3}}}{\varphi_0 K}\right)^{\frac{1}{1-n}} \qquad (26)$$

$$\text{or } v=A^{\frac{1}{1-n}}(\varphi_0 K)^{\frac{n}{1-n}}S^{\frac{1}{3(1-n)}}\Delta^{\frac{1}{3}\left(\frac{3n-1}{1-n}\right)} \qquad (27)$$

As is shown by the equation (27), v is independent of the diameter of the powder core. With the substitution of $K=\frac{20.7}{\sqrt{f}}$ into (27), we obtain

$$v=\left\{20.7^{-n}\varphi_0^{-n}A_s^{\frac{1}{3}}\Delta^{(n-\frac{1}{3})}f^{\frac{n}{2}}\right\}^{\frac{1}{1-n}} \qquad (27)'$$

For example, if we assume $n=0.5$, $h=2$ for $A=CT^h$, $f=0.38\,V_oT$ where V_o is the specific volume of gas at normal temperature and pressure, we find

$$v=\frac{3\times10^{-2}}{\varphi_0}C^2V_o^{0.5}T^{4.5}S^{0.67}\Delta^{0.24} \qquad (27)''$$

The relation (27)″ gives estimation on the effects of various factors such as, temperature of combustion T, real density of powder S, frictional coefficient φ_0, specific volume of gas V_o, and the loading density Δ. φ_0 depends not only on the structure of a fuse but also on the nature and amount of solid residue after burning, and the adhesive property of this residue against the inside wall of fuse threads. Moreover, in a commercial safety fuse, core threads and inside threads of a fuse contribute some amounts of gases to the total amount of gaseous products which are produced from the powder proper. These corrections must be taken into account by modifying the value of the coefficient φ_0.

VI. Burning velocity of safety fuse under laterally complete tamping.

In practical use, it is more common that a safety fuse is ignited while it is laterally tamped in a bore hole, the completeness of tamping fluctuating from case to case. In this case, the burning occurrs no more under stationary condition but accelerating is realized, therefore, the solution of the problem is practically important.

In complete tamping q tends to zero, therefore, the amount of discharge per unit time at the length of a fuse x

$$G_x=\frac{\varphi_0 F_0 Kp}{1+bx} \qquad (28)$$

In equation (28), the value of p varies with the distance x. On the other hand, the amount of gas produced by burning is

$$G_c=F_0Ap^n\left(\frac{S}{\Delta}\right)^{\frac{1}{3}}\Delta=F_0Ap^nS^{\frac{1}{3}}\Delta^{\frac{2}{3}} \qquad (29)$$

Although the value of p in (29) varies with the distance x, the amount (28) corresponds with that of (29) at arbitrary x, therefore we get

$$\frac{\varphi_0F_0Kp}{1+bx}=F_0Ap^nS^{\frac{1}{3}}\Delta^{\frac{2}{3}}$$

$$\text{or } p=\left(\frac{AS^{\frac{1}{3}}\Delta^{\frac{2}{3}}}{\varphi_0 K}\right)^{\frac{1}{1-n}}(1+bx)^{\frac{1}{1-n}} \qquad (30)$$

$$v=\frac{dx}{dt}=Ap^n\left(\frac{S}{\Delta}\right)^{\frac{1}{3}}$$

$$=(\varphi_0K)^{\frac{n}{n-1}}\left\{AS^{\frac{1}{3}}\Delta^{\frac{1}{3}(3n-1)}(1+bx)^n\right\}^{\frac{1}{1-n}}$$

$$=\left\{A^{\frac{1}{1-n}}(\varphi_0K)^{\frac{n}{1-n}}S^{\frac{1}{3(1-n)}}\Delta^{\frac{1}{3}\left(\frac{3n-1}{1-n}\right)}\right\}\cdot(1+bx)^{\frac{n}{1-n}} \qquad (31)$$

If we denote the value of v at $x=0$, by v_0, then v_0 means the burning velocity in free air without lateral tamping.

$$v_x=v_0(1+bx)^{\frac{n}{1-n}} \qquad (32)$$

If $n=0.5$, then

$$v_x = v_0(1+bx) \qquad (33)$$

In practice, the total time of combustion t_t is important for which we obtain

$$t_t=\int_0 \frac{dx}{v_0(1+bx)^{n/(1-n)}}=\frac{(1-n)}{(1-2n)}\frac{1}{bv_0}\cdot \left\{(1+bL)^{\frac{1-2n}{1-n}}-1\right\} \qquad (34)$$

If $n=0.5$, then

$$t_t=\int_0^L \frac{dx}{v_x}=\frac{1}{v_0 b}ln(1+bL)=\frac{2.303\log(1+bL)}{v_0 b} \qquad (35)$$

As we are able to find the value of b by the method described in IV, we can find the value of n by plotting $t_t \sim L$ for a given value of b trying various assumed values for n which are to be in the range between 1 and zero. The numerical value of t_t in (34) is very sensitive to the minute change of n, therefore, we are able to determine the exponent n with considerable accuracy.

Experimental

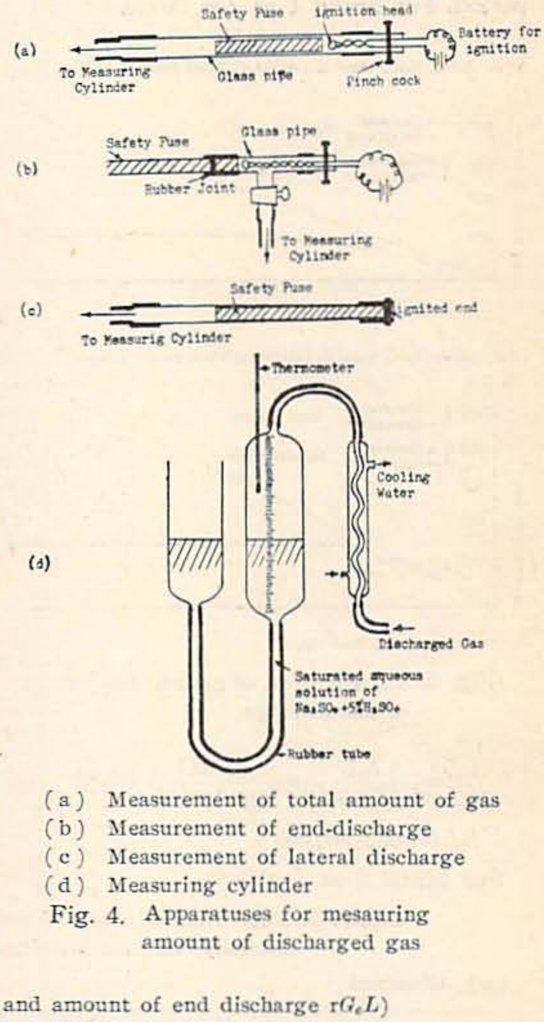
I. Experimental procedure.

The apparatuses used in the experiment for measuring the amounts of discharged gas from a fuse are shown in Fig. 4. (a), (b), (c) and (d).

The procedure employed is very simple and it may easily be understood from the illustration in Fig. 4.

II. Empirical Results and Calculation for amounts of gas.

Empirical Results obtained for the two kinds of commercial safety fuses, brand 1 and 2, are tabulated in Table 1 and plotted in Fig. 5.



(a) Measurement of total amount of gas
(b) Measurement of end-discharge
(c) Measurement of lateral discharge
(d) Measuring cylinder

Fig. 4. Apparatuses for mesauring amount of discharged gas

Table 1. Total amount of gas (G_tL) and amount of end discharge rG_eL)

Length of safety fuse cm		5	10	15	20	30	40	50	60
Brand 1.	G_tL c. c. (mean)	67, 64 64 (65)	133, 126 (130)	.	290, 300 310(303)		580, 535 555(558)		
	G_eL		108		195, 177 170(181)	233	240	250	253
Brand 2.	G_tL		207, 195 (201)		380, 380 400(387)	575, 555 (565)			
	G_eL		183	238	330, 340 320(330)	350	353		345

Fig. 5. shows that for brand 1 after $L=x_s=30$ cm. (for brand 2, $L=x_s=20$cm.) the amount of end discharge tends to a constant value, therefore for brand 1, $x_s=30$ cm is the boundary between end-discharge period and lateral-discharge period. For brand 1 at $L=30$ cm

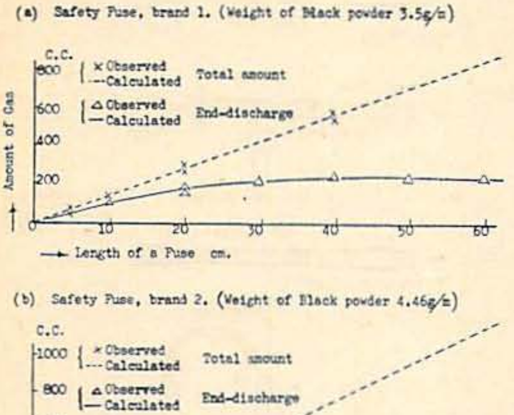


Fig. 5. Total amount of gas and amount of end-disharge

$$\frac{G_tL}{G_eL}=\frac{b_1L}{ln(1+b_1L)}=\frac{0.4343}{\log(1+b_1L)}=1.80$$

$$b_1L=1.95, b_1=0.065$$

For brand 2 at $L=20$cm.

$$\frac{G_tL}{G_eL}=1.21,\quad b_2L=0.30,\quad b_2=0.015.$$

The smooth curves shown in Fig. 5. are calculated by the following equation

$$G_eL=(\varphi_0F_0Kp)\left(L-\frac{2.303}{b}\log(1+bL)\right) \quad (21)$$

while the value of (φ_0F_0Kp) is calculated by the followiug relation

$$(\varphi_0F_0Kp)=\frac{G_tL}{L} \quad (20)$$

It may be seen from Fig. 5. that agreement between the observed and calculated results is good.

Ⅲ. Empirical results for burning with lateral tamping and calculated results.

Safety fuse brand 1 was insulated against lateral discharge of gas by covering tightly the side of it with insulating tape for electric code and thin copper wire. To ascertain that there be no lateral escape of gas, the total amount of gas discharged axially from the ignited end was also measured. For various total lengths of the safety fuse, brand 1, the relation between the total time of combustion t_t and the total length of the fuse L was measured for this state of laterally complete tamping, the results being shown in Table 2 and Fig. 6.

The theoretical relation between t_t and

Table. 2. Total time of combustion t_t and total length of a fuse L with laterally complete tamping.

(a) Observed

Length of fuse L cm	10	20	30	40	50	60	70	80	90	100	with no tamping
Total time of combustion sec	12.4	22.2	1	36.7	44.7	51.8	57.3	64.0	–	80.5	122.1
(Total amount of gas) c. c.	(158)	(325)	(485)	(640)	(834)	(970)	–	(1,300)	–	(1,510)	–

(b) Calculated

for $n=0.333$ ($m=0.5$)	10.7	19.2	26.6	33.2	39.2	44.7	50.2	55.0	59.8	64.2	
for $n=0.231$ ($m=0.7$)	11.1	20.9	29.8	38.3	46.2	53.9	61.2	68.4	75.2	81.9	

L is as follows:

$$t_t=\frac{(1-n)}{(1-2n)}\frac{1}{bv_0}\left((1+bL)^{\frac{1-2n}{1-n}}-1\right) \quad (34)$$

By the measurement of total time of burning of a fuse for a given length

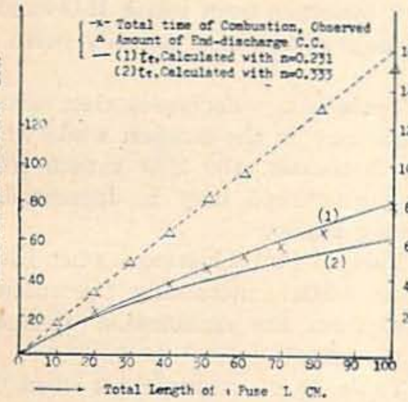


Fig. 6. Determination of the exponent n by the relation between total time of combustion and total length of a fuse L with laterally complete tamping

($L=100$ cm in this example) without lateral tamping, we obtained

$$v_0=\frac{100\ \text{cm}}{122\ \text{sec}}=0.833\ \text{cm/sec}.$$

With substition of assumed values for $m=\frac{1-2n}{1-n}$ into (35), we calculate the relation between t_t and L with the value $b=0.065\text{cm}^{-1}$

The calculated results are shown in Table 2 and in Fig. 6. The results indicate that $n=0.231$ is the most probable.

Discussion

I. Stability of combustion

In practical use, the stability of combustion is one of the most important requisites for safety fuses.

The amount of gas G_t discharged per unit time is represented as a function of pressure p by equation (17)

$$G_t=(\varphi_0 F_0 K)=C_1 p \quad (17)'$$

While the amount of gas evolved at combustion front by burning of a powder per unit time is

$$G_c=(F_0\Delta v)=(F_0 \Delta S^{\frac{1}{3}} \Delta^{\frac{2}{3}})p^n=C_2 p^n \quad (36)$$

Steady burning can be realized at pressure p_s at the combustion front, p_s being determined by the crossing point p_s of the straight line (17)′ and a curve (36) concave downward as are shown in $G\sim p$ relations in Fig. 7.

If any small disturbance causes an increase of this stationary pressure p_s, say, to $p_s+\delta p$, then the rate of gas evolution by burning will be increased by $\delta G_c=nC_2p^{n-1}\delta p$ (37) the relation being found easily by differentiating (36) with respect to p, while the amount of discharge will be increased by

$$\delta G_t=C_1\delta p \quad (38)$$

The index for the stability of combustion may be represented by the speed of readjustment of this disturbed pressure $p_s+\delta p$ to the original p_s. This speed Ad, may be assumed to be proportional to the difference between $(\delta G_t-\delta G_c)$

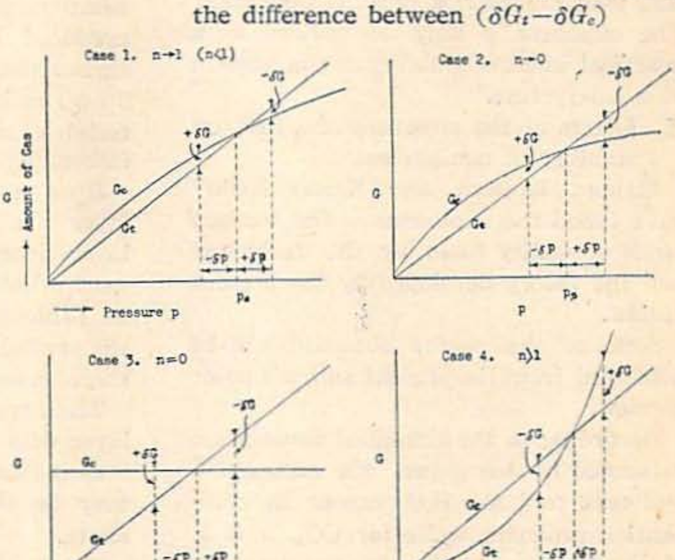


Fig. 7. Stabillity of combustion for various n.
G_c=rate of gas evolution by burning of powder c. c./sec.
G_t=rase of total gas discharge c. c./sec.

$$Ad = \text{const.}\ (\delta G_t - \delta G_e) \qquad (39)$$

If the sign before (39) is the opposite to that of the disturbance on pressure, $+\delta p$, then, the stationary pressure can again be established, while in an opposite case any small disturbance will continue to enlarge leading to an accelerating combustion, or in case of negative disturbance $(-\delta p)$ leading to fading out of combustion.

Fortunately, in practice, the sign of Ad is the opposite to that of a given disturbance δp as is shown in Fig. 7, case (1), (2), (3), while the case (4), where the exponent n of burning velocity is larger than 1, does not exist.

$$Ad = \text{const.}\ (C_1 - nC_2 p^{n-1})\delta p \qquad (40)$$

It is obvious that for the case (3) $n=0$ the stability is the best, while for the case (2), where n is near zero, the stability is fairly good. In the case (1) where $n \to 1$ the stability tends to worse, therefore, in practice, we should design a safety fuse so as to realize a smaller value of n, and, if possible, n should be zero which is the case of a "gasless fuse." The exponent n may be utilized as a practical index of stability of combustion of a safety fuse.

II. Effects of the structure of a fuse on stability of combustion.

Makoto Kimura and Kazuo Kudo[4] have found the exponents n for various kinds of safety fuses by the technique and the theory developed by the present author.

Some of the results obtained will be discussed from the present author's point of view.

According to the simplified assumption presented in this paper, the exponent n will tend to $\frac{4}{5}$ for H_2O vapour in combustion products, while for CO_2, n is $\frac{1}{3}$. If the energy tranfer to the unburnt powder from the combustion products is mainly through solid residue, then the exponent n will tend to zero.

Black powder in a safety fuse does not produce H_2O because it consists of potassium nitrate KNO_3, sulphur S and charcoal (mainly C) while in a commercial safety fuse the core threads and the innermost threads of a fuse decompose and burn with the powder, therefore, the "effective powder" include, to some extent, hydrogen from which H_2O vapour could be produced in a quantity more or less.

In Table 3 (a) n decreases when carbon becomes rich in the powder, while when KNO_3 in-creases, the H_2O vapour from hemp yarn thread may be increased, n becoming bigger.

In Table 3 (b) n increases when KNO_3 becomes richer, increasing the amount of H_2O from the combustion of hemp yarn threads.

In Table 3 (c) n increases when the degrees of carbonization of charcoal decrease, increasing the contents of hydrogen in charcoals.

In Table 3 (d) n increases when the weight of powder per meter of a fuse increases, increasing the amount of hemp yarn burnt by the powder.

Paper thread is more easily burnt than hemp yarn, therefore, the amount of H_2O produced is greater in case of paper thread than in case of hemp yarn. Table 3 (e) shows that n increases as the materials of threads change from hemp yarn into paper.

If we change the materials of the second layer of a fuse (second to the innermost layer) from paper into cotton, the exponent n indicates no nhange as is shown in Table 3. (b), the result showing that the second layer does not interfere with the combustion of the powder itself.

The covering by asphalt on the second layer does not also affect the exponent n as is shown in Table 3. (g), the reason may be the same with that described above.

If we change the innermost threads into non-combustible glass-wool yarn, then only the black powder burns with no H_2O and with rich carbon and solid residue, thus the exponent n will become much smaller. Table 3. (h) shows that this is the case.

Table 3. Relation between the exponent n and the structure of safety fuses.

(a) Effect of KNO_3 (S constant)

KNO_3	S	C	Exponent n
45	20	35	0.043
50	20	30	0.0504
55	20	25	0.060
60	20	20	0.124
65	20	15	0.126
70	20	10	0.139

(b) Effect of KNO_3 (C constant)

KNO_3	S	C	n
45	25	30	0.0328
50	20	30	0.0504
55	15	30	0.028
60	10	30	0.063
65	5	30	0.097

(c) Effect of carbon content

Index of carbonization	n	
30.06	0.016	degree of carbonization is in the reverse order as in the order of index of carbonization. The composition (KNO_3 45% S 20% C 35%)
40.10	0.032	
50.00	0.050	
59.41	0.053	

(d) Effect of amount of powder per meter of a fuse

Composition			Amount of powder	n
KNO_3	S	C		
45	20	35	3.2 g/m	0.031
			4.5	0.045
			5.6	0.049
60	25	15	3.4	0.130
			5.0	0.191
			5.6	0.219
62	18	20	3.5	0.157
			4.8	0.218
			6.0	0.325

(e) Effect of innermost threads

Composition			Material of innermost threads	n
KNO_3	S	C		
62	18	20	Hemp yarn 13 threads	0.2039
			Hemp yarn 7 paper 6	0.225
			paper 6	0.241

(f) Effect of second layer

Compostion			Material of second threads	innermost threads	n
KNO_3	S	C			
45	20	35	paper 8	Hemp yarn 13	0.031
〃	〃	〃	cotton 6	〃	0.030
50	20	30	paper 8	〃	0.045
〃	〃	〃	cotton 6	〃	0.045

(g) Effect of asphalt covering second layer

Composition			Asphalt	n
KNO_3	S	C		
62	18	20	none	0.128
〃	〃	〃	thin	0.160
〃	〃	〃	thick	0.175
50	15	35	none	150.0
〃	〃	〃	thick	0.014
45	20	35	none	0.030
〃	〃	〃	thick	0.036

(h) Effect of non-combustble innermost layer (Glass-wool yarn)

Kind	Innermost layer	second layer	n
C_1	glass wool yarn 13	paper thread 8	0.0084
C_2	〃 13	〃 8	0.0198

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