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SHOCK WAVE THEORY OF BLASTING WITH CYLINDRICAL CHARGE

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Summary

Shock wave theory of blasting with a long cylindrical charge has been described along the same lines as done in a series of previous papers by the present author on the shock wave theory of blasting with concentrated charge and its applications to practical problems.

Practical formulas have been summarized as follows:

(1) Peak pressure p_r of a cylindrical shock wave at distance r from an axis of a cylindrical charge whose radius is "a":

$$p_r=p_D\left(\frac{a}{r}\right)^n$$

where p_D=detonation pressure

$D=\frac{r}{a}$=reduced distance n=1.5

(2) Peak pressure p_a on the free face:

$$p_a=p_D\left(\frac{a}{d}\right)^{1.5}$$

where d=depth to a charge or burden to be blasted

(3) Peak pressure p_{1-2} at the second free face produced by the first slab:

$$p_{1-2}=p_a-S_t$$

where S_t=tensile strength of rock

(4) Mean pressure p_1 of a shock wave entrapped within the first slab:

$$p_1=p_D\left(\frac{a}{d}\right)^{1.5}-\frac{S_t}{2}$$

(5) Velocity of outward movement V_1 of the first slab due to momentum given by a shock wave:

$$V_1=\left(\frac{2g}{\Delta_r U}\right)P_1$$

where g=gravity constant

Δ_r=density of rock

U=shock wave velocity

(6) Depth of a crater d_f:

$$d_f=\frac{L}{2}$$

where L=effective length of a shock wave

(7) Coefficient of pressure decay n: Method I.

$$n=\frac{\log\left\{\left(\frac{p_D}{S_t}\right)\left(\frac{d^2}{R^2+d^2}\right)\right\}}{\log\left(\frac{\sqrt{R^2+d^2}}{a}\right)}$$

where R=radius of a crater (half of the breadth of a trough shape crater)

(8) Burden d_f for a full crater:

$$S_t=p_D\left(\frac{a}{2d_f}\right)^{1.5} \quad \text{or} \quad d_f=\frac{a}{2}\left(\frac{p_D}{S_t}\right)^{\frac{1}{1.5}}$$

Reduced burden $D_f=\frac{d_f}{a}$

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(9) Coefficient of pressure decay n: Method II.

$$n=\frac{\log\left(\frac{p_D}{S_t}\right)}{\log\left(\frac{2d_f}{a}\right)}$$

(10) Time required t_s for slab formation due to a shock wave

$$t_s=\frac{2d_f}{U}$$

(11) Displacement of stemming x after time t from the moment of detonation

$$x=\sqrt{x_0^2+\left(\frac{p_D v_D}{M}\right)t^2}$$

where x_0=original length of a charge, v_D=volume of a charge, M=mass of stemming

(12) Velocity of the second slab V_2:

$$V_2=\left(\frac{2g}{\Delta_r U}\right)\left(p_d-\frac{3}{2}S_t\right)$$

(13) Velocity of the third slab V_3:

$$V_3=\left(\frac{2g}{\Delta_r U}\right)\left(p_d-\frac{5}{2}S_t\right)$$

(14) Time t_b at which detonation products begin to blow out of a borehole:

$$t_b=\sqrt{\frac{M}{p_D v_D}(l_h^2-l_e^2)}$$

where l_h=total length of a bore hole

l_e=total length of a cylindrical charge$=x_0$

(15) Spacing S_e between bore holes: $S_e=xd_f$, $x=S_e/d_f$=reduced spacing,

$$x=2\sqrt{2^{\frac{2}{n+2}}-1} \quad \text{for } n=1.5 \quad x=1.4$$

(16) Length of sub-drilling l_s:

$$l_s=\frac{4}{3}\left(\frac{S_t}{p_D}\right)^{\frac{1}{n}}d_f\fallingdotseq(0.35d_f)$$

(17) Length of stemming near a mouth of a bore hole l_n:

$$l_n=2\left\{1-\frac{2}{3}\left(\frac{S_t}{p_D}\right)^{\frac{1}{n}}\right\}d_f\fallingdotseq(1.65d_f)$$

(18) Loading factor for a cylindrical charge

$$F=2.86\times10^6\pi\left(\frac{\Delta_e}{\Delta_r}\right)\left(\frac{S_t}{p_D}\right)^{1.33}\ \frac{\text{gram explosive}}{\text{ton of rock}}$$

where Δ_e=loading density of an explosive charge

(19) Loading factor for a concentrated charge (coyote blasting)

$$F=4.05\times10^6\pi\left(\frac{\Delta_e}{\Delta_r}\right)\left(\frac{S_t}{p_D}\right)^{1.5}$$

(20) Loading factor for a deck charge

$$F=6.31\times10^6\pi\left(\frac{\Delta_e}{\Delta_r}\right)\left(\frac{S_t}{p_D}\right)^{1.5}$$

(21) Number of slabs N:

$$N=\frac{p_d}{S_t}=2^n$$

(22) Thickness of a slab l:

$$l=\frac{d_f}{N}=\frac{a}{2^{n+1}}\left(\frac{p_D}{S_t}\right)^{\frac{1}{n}}$$

(23) Number of bore holes B per 100m² for cylindrical charges:

$$B=\frac{2.86\times10^6}{a^2}\left(\frac{S_t}{p_D}\right)^{1.33}$$

(24) Number of bore holes B per 100m² for deck charges:

$$B=\frac{2.33\times10^6}{a^2}\left(\frac{S_t}{p_D}\right)$$

Introduction

For the benefit of the simplicity of the theoretical treatment the theory of blasting of solid rocks may be divided into two extreme cases, that is, the theory of blasting with a concentrated charge and theory of blasting with a cylindrical charge. The former case has been dealt with in the previous[1] papers while the present paper is an attempt to establish a shock wave theory of blasting with a cylindrical charge.

The definition of terms in blasting with a cylindrical charge has been illustrated in Fig. 1.

In the present theoretical treatment it has been assumed that the length of a cylindrical charge is long enough compared with the burden and the detonation velocity of the column of an explosive is infinitely fast, that is, the wave front of the shock wave produced by the detonation of the charge forms a cylindrical wave which propagates two-dimentionally except near the both ends of a columnar charge.

§ 1. **Outline of the theory**

The blasting of a solid material by an explosive charge may occur in the following sequence:

(1) Detonation of an explosive charge produces "crushed zone" around it so far as the intensity of shock wave is greater than the compressive strength of the rock, however, this range of crushing is limited to the neighbourhood of a charge.

(2) Beyond the crushed zone there can be no breaking by compression due to shock wave, however, the shock wave is reflected as a tension wave at a free face. As the tensile strength of rock is much smaller than compressive one, rock can be broken by this tension wave, the range

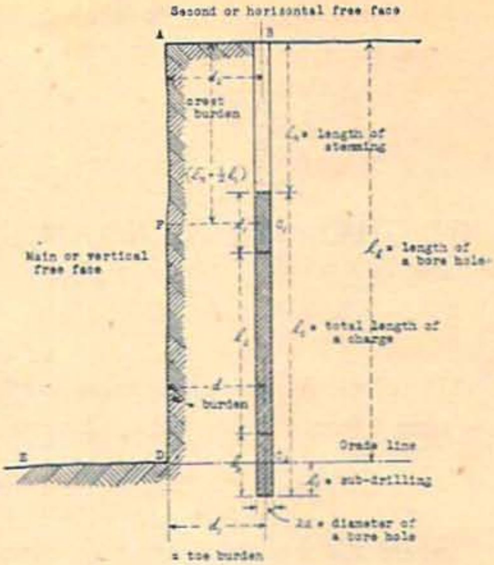

Fig. 1. Definition of terms in blasting with a cylindrical charge

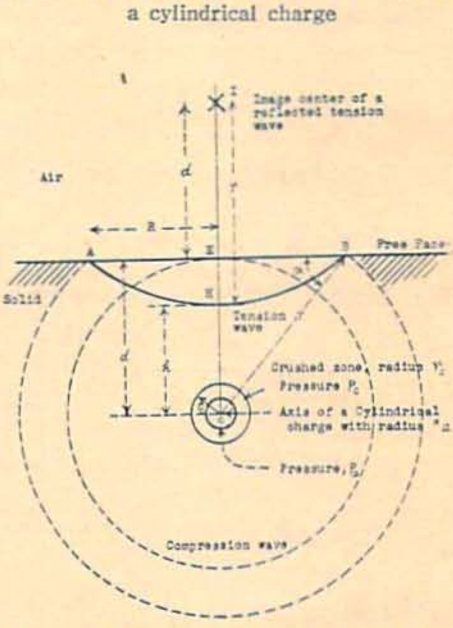

Fig. 2. Propagation of a cylindrical shock wave (section) and its reflection at a free face

of breaking extending from a free face inwards to the crushed zone.

(3) Only a part of the total energy of an explosive charge goes into a shock wave and the gases from detonation at high pressure expand doing work againt

1) Kumao Hino: Theory of blasting with concentrated charge: This Journal, **15**, No. 4, 233~249, (1954) and Kumao Hino: Fragmentation of rock through blasting and shock wave theory of blasting. Symposium on Rock Mechanics, Quarterly of the Colorado School of Mines, **51**, No. 3, 191. (1956)

fragments of rock which have been produced by a shock wave. These fragments are already in motion due to momentum acquired from a shock wave and they get more momentum from the expansion of gases.

§ 2. Characteristics of a shock wave in rock

Detonation pressure p_D of an explosive may be calculated by the hydrodynamical-thermodynamical theory of detonation or by some approximation formulas[2].

Characteristics of a shock wave within rock may be calculated[1] on the basis of Riemann, Rankine-Hugoniot's equations of a shock wave and an equation of state of solid, however, a strong shock wave may exist only within "a crushed zone" and beyond this crushed zone the intensity of shock wave is weaker than compressive strength of rock, moreover, the intensity of shock wave decreases rapidly due to two dimentional divergence of the shock wave. Because of these circumstances in addition to the fact that compressibility of solid rock is generally very small, an acoustic approximation may be able to explain fairly well the behaviours of shock waves and this situation makes further theoretical treatment much easier.

§ 3. Pressure decay due to distance

Outside of a crushed zone we may assume that the peak pressure p_r of a shock wave at a distance r from an axis of a cylindrical charge, whose radius is "a", be represented by the following equation:

$$p_r=p_D\left(\frac{a}{r}\right)^n \qquad \text{(1)}$$

or $$p_r=p_D\left(\frac{1}{D}\right)^n \qquad \text{(1)}'$$

where: p_D=detonation pressure

D=reduced distance$=\frac{r}{a}$

The numerical value of n may be assumed to be 1~2 while it should depend on the nature of rock and an explosive.

§. 4. Breaking by a reflected tension wave

4-1. Formation of slabs by a tension wave.

In Fig. 2. C is a section of an axis of a cylindrical charge whose radius is "a". The pressure at "a" at the moment of detonation of an explosive charge, whose length is assumed to be infinite, may be assumed to be equal with the detonation pressure p_D. r_C indicates a radius of a crushed zone and the peak pressure of a shock wave at r_C may be assumed to be the same with the compressive strength of rock p_C. At a free face AB which is situated at a distance d from an axis of a cylindrical charge the compressive shoch wave reflects as a tension wave. At a point H where the effective tension, that is, the difference between the intensity of a tension wave and that of the remaining compression wave, exceeds the tensile strength of rock, whose value it much smaller than the compressive strength, the first main fracture due to tension takes place at H and the portion of rock between H and a free face is defined as the first slab. If we describe the peak pressure of a shock wave at a point E on a free face, by p_d then we have:

$$p_d=p_D\left(\frac{a}{d}\right)^n \qquad \text{(2)}$$

To a first approximation the pressure

2) Kumao Hino and T. Urakawa: Approximate detonation pressure of industrial explosives: This Journal, 7 No. 4. 242~250 (1956)

of a shock wave at H where the first main fracture due to tension takes place may be described as follows:[3]

$$p_{1-2}=p_d-S_t \quad \cdots\cdots\cdots\cdots\cdots\cdots(3)$$

where S_t=tensile strength of rock.

Then the mean pressure of shock wave entrapped within the first slab is as follows:

$$p_1=\frac{p_d+p_{1-2}}{2}=p_D\left(\frac{a}{d}\right)^n-\frac{S_t}{2}\cdots(4)$$

The outward velocity of the first slab at a point E on a free face may be represented as follows:[4]

$$V_1=\left(\frac{2g}{\Delta_r U}\right)p_1\cdots\cdots\cdots\cdots\cdots\cdots(5)$$

where: g=gravity constant
Δ_r=density of rock
U=velocity of a shock wave

In this way the first slab $E\ H$ moves outwards away from the axis of a cylindrical charge and a new free face $A\ H\ B$ is produced. So far as the intensity of the remaining compressive shock wave exceeds the tensile strength of rock the similar process repeats itself, thus producing the second slab, the third slab and so on.

If we define the effective length of a shock wave L to be a length of a shock wave whose intensity exceeds the tensile strength of rock then the depth of a crater d_f may be represented by the following equation.

$$d_f=\frac{L}{2} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots(6)$$

3) Kumao Hino: Fragmentation of rock through blasting: This Journal, 17, No. 1. 2-11. (1956)

4) Kumao Hino: Velocity of rock fragments and shape of shock wave: ibid. 17. No. 4, 236~241. (1956)

4-2. Estimation of decay constant n.
The method I.

In Fig. 2. at the periphery of a crater A or B the outwards component of the intensity of a shock wave vertical to a free face may be assumed to be the same with the tensile strength of rock.

Then we have:

$$\sin\alpha=\frac{d}{r}=\frac{d}{\sqrt{R^2+d^2}}\cdots\cdots\cdots(7)$$

$$S_t=p_r(\sin\alpha)^2 \quad \cdots\cdots\cdots\cdots\cdots\cdots(8)$$

$$\text{or } S_t=p_D\left(\frac{a}{r}\right)^n(\sin\alpha)^2\cdots\cdots\cdots(8)'$$

$$\text{or } S_t=p_D\left(\frac{a}{r}\right)^n\frac{d^2}{R^2+d^2}$$

$$=p_D\left(\frac{a}{\sqrt{R^2+d^2}}\right)^n\frac{d^2}{R^2+d^2}\quad\cdots(9)$$

$$\text{then}\left(\frac{R}{d}\right)^2+1=\left(\frac{p_D}{S_t}\right)^{\frac{2}{n+2}}$$

$$\left(\frac{a}{d}\right)^{\frac{2n}{n+2}}\cdots\cdots\cdots\cdots\cdots\cdots\cdots(10)$$

from (8)′ we have:

$$n=\frac{\log\left\{\left(\frac{p_D}{S_t}\right)(\sin\alpha)^2\right\}}{\log\left(\frac{r}{a}\right)}$$

$$=\frac{\log\left\{\left(\frac{p_D}{S_t}\right)\left(\frac{d^2}{R^2+d^2}\right)\right\}}{\log\left(\frac{\sqrt{R^2+d^2}}{a}\right)}\quad\cdots(11)$$

The following data have been obtained for limestone (Joban Mining Co. Kokura Quarry in Kyushu, Japan):

l_h=length of bore hole drilled parallel to a vertical free face=4.5 meter
l_c=total length of a continuous cartridge=2.7 meter
W=total weight of the charge=3.750kg
$2a$=diameter of a bore hole=3.4 centimeter, a=1.7cm
d_f=burden for a full crater=1.5meter
R=half of breadth of a ditch-shaped

crater.

$R = AE$ in Fig. 2=2.00m

S_t=tensile strength of rock assumed[3] to be 55kg/cm².

U=velocity of shock wave within rock assumed [4] to be 5,740m/sec.

Δ_r=density of rock 2.63g/cm³

p_D=detonation pressure of Ammon Gelatin (Shin Kiri Dynamite) assumed[2] to be 160×10^3kg/cm².

Δ_e=loading density 1.45g/cm³.

The above case is often referred to in the present paper and it may be described in the following as the model case.

Then we have:

$$r=\sqrt{R^2+d^2}=\sqrt{200^2+150^2}=250\text{centimeter}$$

$$(\sin\alpha)^2=\left(\frac{d}{r}\right)^2=\left(\frac{150}{250}\right)^2=0.360$$

$$n=\frac{\log\left(\frac{160\times10^3\times0.360}{55}\right)}{\log\left(\frac{250}{1.7}\right)}$$

$$=1.394\fallingdotseq1.4$$

4-3. Reduced burden

Reduced distance or reduced burden for a full crater is:

$$D_f=\frac{d_f}{a} \quad\text{.............................(12)}$$

$$\text{or } D_f=\frac{150}{1.7}=88.2$$

This numerical value of reduced burden is a constant for a pair of rock and explosive and is important in the design of blasting. On the basis of this value we can easily estimate the burden for a different radius of a cylindrical charge or vice versa. For example if we use the radius of a cylindrical charge a=5cm, then the burden should be:

$$d_f=D_f a=88\times5=440\text{cm}$$

It must be remembered that this important relation holds theoretically only in the case of an infinitely long cylindrical charge with a free face parallel to the axis of a cylindrical charge. Similar important similitude relation holds in the case of a spherical charge[1] while in the case of a spherical charge radius "a" indicates a radius of a spherical charge. In practice intermediate cases may be often realized. Roughly speaking when the total length of a charge "l_c" exceeds the burden "d" then the charge may be assumed to be "cylindrical" while in the case where the total length of a charge l_c is shorter than a burden d it may be assumed to be "spherical" because a shock wave produced from a short cylindrical charge changes its form from a cylindrical one into a spherical one as the distance of propagation increases and this degree of expansion may be expressed by the relative dimension of the burden compared with the total length of an explosive charge.[5] In previous literatures on blasting these two extreme cases of "a concentrated charge" and "a cylindrical charge" have been confused not only in theoretical treatments but also in practical applications and this important relation of "reduced burden $D_f=\frac{d_f}{a}$" which is quite useful has been misused. If we can carefully discriminate between these two cases most of the data described in previous publications on blasting may be usefully analysed und be utilized from the standpoint of the shock wave theory of

5) Kumao Hino: Concentrated type of no-cut round of blasting: ibid. 16, No. 3, p. 49, Fig. 9. (1955)

blasting.

4-4. Estimation of decay constant n. The method II.

Another approximate relation for a full crater may be obtained as follows on the basis of a hypothesis of a single reflection. In Fig. 2. the front of a reflected tension wave H must reach the axis of a cylindrical charge C to realize a full crater, then, the intensity of a shock wave should be equal with the tensile strength of rock after having travelled over a distance $2d$ and we have the following relation for a full crater condition.

$$S_t = p_D\left(\frac{a}{2d_f}\right)^n \quad \text{.................. (13)}$$

For a full crater we get the following relation:

$$\text{Reduced burden } D_f = \frac{d_f}{a} = \frac{1}{2}\left(\frac{p_D}{S_t}\right)^{\frac{1}{n}} \quad \text{..................... (14)}$$

whose value is a constant for a pair of rock and explosive irrespective of the absolute values of burden d_f or radius of a cylindrical charge "a".

From the equation (13) we may find the value of n as follows:

$$n = \frac{\log\left(\frac{p_D}{S_t}\right)}{\log\left(\frac{2d_f}{a}\right)} \quad \text{..................... (14)}$$

or in the case above mentioned:

$$n = \frac{\log\left(\frac{160\times10^3}{55}\right)}{\log\left\{\frac{2\times150}{1.7}\right\}} = 1.54$$

In the following calculations the value of n may be assumed to be $n=1.5$

Once we have known the numerical value of n then we may calculate the pressure of a shock wave at the original free face p_a, number of slabs N, and velocity of the first slab V_1 as follows:

$$p_a = p_D\left(\frac{a}{d}\right)^n = 160\times10^3\left(\frac{1.7}{150}\right)^{1.5} = 192\text{kg/cm}^2$$

$$N = \frac{p_a}{S_t} = \frac{192}{55} \fallingdotseq 3.5 \fallingdotseq 3\sim4$$

$$V_1 = \frac{2g}{\Delta_r U}p_1 = \left(\frac{2\times980}{2.63\times574000}\right)\left(192-\frac{55}{2}\right) = 1.298\times164.5 = 213\text{cm/sec.}$$

4-5. Estimation of tensile strength of rock from reduced burden

If we know the detonation pressure p_D of an explosive and the tensile strength of rock S_t then we can calculate the reduced burden D_f by the equation (14):

$$D_f = \frac{d_f}{a} = \frac{1}{2}\left(\frac{p_D}{S_t}\right)^{1.5} \quad \text{......... (14)}$$

For the model case:

$$D_f = \frac{1}{2}\left(\frac{160\times10^3}{55}\right)^{\frac{1}{1.5}} = 85$$

The above equation may be used to estimate practical tensile strength of rock because, we can calculate the detonation pressure p_D at least approximately while we can determine the value of D_f or d_f by blasting experiments. For example R. Westwater[6] has given the following values for diameters of holes and burdens with Polar Ammon Gelignite for any particular rocks of three degrees of strength.

We have from (14):

$$S_t = \left(\frac{1}{2D_f}\right)^{1.5} p_D \quad \text{..................... (15)}$$

6) R. Westwater: Mine & Quarry Engineering: July 1956, p. 285.

Table 1. Estimation of tensile strength from reduced burden

No.	Diameter of bore hole $2a$	Burden in ft. $=d_f$		
1	11 inch	35.2	44.0	52.5
2	$1\frac{1}{4}$ inch	4	5	6
3	Reduced burden d_f/a (Calculated)	76.8	96.0	115.3
4	Tensile strength S_t (Calculated)	81.3kg/cm²	56.6kg/cm²	44.2kg/cm²

p_D may be assumed to be 155×10^3kg/cm². (reference (2)).

Calculated tensile strengths of three kinds of rocks shown in Table 1. (the last line) may be considered to be within reasonable range.

§ 5. **Work done by expansion of gases**

The effects of expansion of gases produced by detonation of an explosive charge may be divided into three phases that is (1) movement of stemming (2) blowing out through a bore hole (3) further expansion of gases which accelerates fragments of rock in part while some of the gaseous products expand through a bore hole and spaces between fragments without doing work of acceleration of fragments. In phase (1) the gases may be assumed to be working within a closed condition while in phases (2) and (3) we must deal with a great amount of leakage of gases.

5-1. Movement of stemming

The time t_r necessary to produce all possible slabs due to action of shock wave may be represented by the following equation:

$$t_s=\frac{2d_f}{U} \quad \text{......(16)}$$

for the model case:

$$t_s=\frac{2\times150}{574000}=0.000522 \text{ sec}$$
$$=0.5 \text{ milli-second.}$$

The weight of stemming W_s

$$=\pi a^2 l_s \Delta_s \quad \text{......(17)}$$

where a=radius of a bore hole

l_s=length of stemming

Δ_s=density of stemming

or

$$W_s=3.1415\times1.7^2\times(450-270)\times2.6$$
$$=4250\text{gram.}$$

In this case we deal with one dimentional motion of the stemming which is assumed to form a rigid piston and an expansion of gases.[7] Let us use the following notations.

E_x=work done by gases at displacement x of the stemming

A_x=work done against the balancing pressure p_s of stemming plus atmospheric pressue.

C_x=velocity of movable mass M

L_x=kinetic energy of the mass M

Then, $E_x=A_x+L_x$ ......(18)

$$E_x=\int_{v_D}^{v_x} p_x dv_x=\int_{v_D}^{v_x}\frac{p_D v_D^k}{v_x^k}dv_x$$
$$=\frac{p_D v_D}{k-1}\left[1-\left(\frac{v_D}{v_x}\right)^{k-1}\right] \quad \text{......(19)}$$

7) C. Ramsauer, Ann. d. Phys. 1923, p. 2650.

where v=volume of gases. v_D=initial volume

k=index of polytropic expansion

$$A_z = p_s(v_z - v_D) \quad \text{......(20)}$$

$$L_z = \frac{M}{2} C_z^2 \quad \text{......(21)}$$

Therefore:

$$\frac{p_D v_D}{k-1}\left[1-\left(\frac{v_D}{v_z}\right)^{k-1}\right] = p_s(v_z - v_D) + \frac{M}{2} C_z^2 \quad \text{......(22)}$$

$$E_{max.} = \frac{p_D v_D}{k-1} \quad \text{......(23)}$$

The present problem is to find the values of v_z and p_z at time t_s. We use the following notation:

$v = \pi a^2 x$, $C_z = \frac{dx}{dt}$ and we may neglect $p_s(v_z - v_D)$ to a first approximation, then, the equation (22) may be written as follows:

$$\frac{dx}{dt} = \sqrt{\frac{2}{M}\left(\frac{p_D v_D}{k-1}\right)\left[1-\left(\frac{x_0}{x}\right)^{k-1}\right]} \quad \text{......(24)}$$

where x_0=length of a charge.

According to H. Jones[8] and A. R. Miller the numerical values of $k=\gamma$ for TNT of specific volume $\left(v=\frac{1}{\Delta_0}\right)$ smaller than 1.136 (or density higher than $\Delta_0=0.88$) is about 3.0. Let us assume $k=3$. The equation (24) may be expressed as follows:

$$\frac{dx}{\sqrt{\frac{2}{M}\left(\frac{p_D v_D}{k-1}\right)\left[1-\left(\frac{x_0}{x}\right)^{k-1}\right]}} = dt \quad \text{......(25)}$$

with $k=3$ we have:

$$\frac{x\,d_x}{\sqrt{\frac{p_D v_D}{M}(x^2 - x_0^2)}} = dt \quad \text{......(26)}$$

If we introduce a new variable: $X = x^2 - x_0^2$, then, $x dx = \frac{dX}{2}$ and we have:

$$\frac{dX}{2\sqrt{\frac{p_D v_D}{M} X}} = dt \quad \text{......(27)}$$

Integration of the equation (27) gives:

$$\frac{1}{\sqrt{\frac{p_D v_D}{M}}} X^{\frac{1}{2}} = t + \text{const.} \quad \text{......(28)}$$

At $t=0$, $x=x_0$, $x=0$, then const.=0. or

$$\sqrt{x^2 - x_0^2} = \sqrt{\frac{p_D v_D}{M}}\, t \quad \text{......(29)}$$

$$\text{or } x = \sqrt{x_0^2 + \left(\frac{p_D v_D}{M}\right) t^2} \quad \text{......(30)}$$

For x_0=270cm, $p_D = 160\times10^3$kg/cm^2 $=160\times10^6$g/cm^2, $v_D = 3.1415\times1.7^2\times270 =$ 2450cm^3

$M = \frac{4250}{980}$ g. t_s=0.000522sec.

we have: $\frac{p_D v_D}{M} = 90\times10^9$

$x = \sqrt{72900 + 24570}$

or $x - x_0$=42cm

As the total length of the stemming is 180cm., 42cm of stemming is blown out at t_s=0.000522sec. At this moment the volume of gases is $v_{E_1} = v_a = 3.1415\times1.7^2\times$ 312 cm^3=2810 cm^3 and the pressure of gases

$$p_{E_1} = p_a = p_D\left(\frac{v_D}{v_a}\right)^{\gamma} \quad \text{......(31)}$$

or $p_{E_1} = p_a = 160\times10^3\left(\frac{270}{312}\right)^3 = 103\times10^3$kg/cm^2

5-2. Blowing out through a bore hole. At $t=t_s$=0.000522 sec. the velocity of the middle point of the last slab which

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8) H. Jones and A. R. Miller; The detonation of solid explosives; Proc. Royal Soc. A. 194, 497. (1948)

is the nearest to the crushed zone is as follows:

$$V_3=\left(\frac{2g}{\Delta_r U}\right)\left(p_3-\frac{5}{2}S_t\right)\text{.........(32)}$$

or

$$V_3=\left(\frac{2\times980\times1000}{2.63\times574000}\right)\left(192-\frac{5}{2}\times55\right)$$
$$=1.298\times55.5=72\text{cm/sec.}$$

At time t_b the stemming is all blown out of a bore hole and the gases begin to expand freely out of this opening, that is, out of a bore hole. The time t_b may be obtained from (29) as follows;

$$t_b=\sqrt{\frac{M}{p_D v_D}(x^2-x_0^2)}\,x$$
$$x=l_h,\ x_0=l_c\text{........................(34)}$$

$$\text{or } t_b=\sqrt{\frac{4250}{980\times160\times10^3\times2450}(450^2-270^2)}$$
$$=\sqrt{\frac{4}{361\times10^9}\times129600}$$
$$=0.0012\text{sec.}$$

Therefore we may assume that just after the shock wave has finished the work of producing slabs the gases begin to blow out of a bore hole. At this moment all of the slabs has been already set in motion outwards due to the momentum given by the shock wave and henceforce the pressure of gases begin to be reduced quite rapidly because gases blow out of a bore hole and also blow through gaps between fragments formed from slabs. Because of these circumstances the effect of expansion of gases on blasting seems to be quite smaller than has been expected to be so far.

The rate of blowing out of a bore hole may be estimated as follows:

If we neglect the increase of the volume due to a crushed zone the volume of gases at $t=t_b$ is as follows:

$$v_b=\pi a^2 l_h\text{ ....(35)}$$

where l_h=length of bore hole

or $v_b=3.14\times1.7^2\times450=4090\text{cm}^3$

pressure of gases at v_b is:

$$p_b=p_D\left(\frac{v_D}{v_b}\right)^k\text{........................(36)}$$

or $p_b=160\times10^3\times\left(\frac{270}{450}\right)^3=38.3\times10^3\text{kg/cm}^2$.

The rate of flowing through an opening with a sectional area A is given by the following equation.

$$Q=\frac{\psi p_b A}{\sqrt{RT_b}}=\psi A\sqrt{\frac{p_b}{v_{bs}}}\text{ ....(37)}$$

where $\psi=0.66$[9] and v_{bs}=specific volume of gas

or $Q=0.66\times3.14\times1.7^2\times$

$$\sqrt{\frac{38.3\times10^3\times1000\times980\times3750}{4090}}$$
$$=1105\text{kg/sec.}$$

The total time of discharge T of gases through a bore hole may be estimated by the following thermodynamical equation.

$$T=\frac{2V_b\, v_{bs}^{\frac{k-1}{2}}}{(k-1)A\sqrt{gk\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}\,p_b^{\frac{1}{2}}\,v_{bs}^{\frac{1}{2}}}\left\{\left(\frac{v_0}{v_{bs}}\right)^{\frac{k-1}{k}}-1\right\}$$

for $k=1.4$, $T=0.0445$sec. and for $k=3$, $T=1.81$sec.

As the velocities of the first and the second slabs are higher than that of the last (third) slab the areas of opening formed by the first and the second slabs are wider than that due to the last slab and the gases expand into outer atmos-

9) J. Corner; Theory of the Interior Ballistics of Guns: New York, John Wiley & Sons, Inc. 1950, p. 258. (Interior Ballastics of Leaking Guns).

phere running through these openings while they may give some of their momentum to the fragments already produced and set in motion by a shock wave. Moreover the expanding gases may go into spaces formed among the first, the second and the last slabs and the pressure acting on fragments may be balanced on both sides of each fragments and this situation may reduce the amount of momentum given to the fragments from the gases. At any rate the part played by an expansion of gases in blasting of rock may be much smaller than it has been assumed to be so far.

We may estimate the velocity of the whole mass of rock detached from the ground as follows. The maximum available energy from an explosive charge is as follows:

$$E_{max.}=\frac{p_D v_D}{k-1} \quad \text{......................(23)}$$

or $E_{max.}=\dfrac{(160\times10^3\times1000\text{g/cm}^2)\times2450\text{cm}^3}{3-1}$

$=1960\times10^7\text{g/cm}$.

Total mass of rock broken

$$M=\frac{Rdl_h \Delta_r}{g} \quad \text{.........................(38)}$$

Then the maximum velocity V of mass M may be as follows:

$$E_{max.}=\frac{1}{2}MV^2 \text{.....................(39)}$$

or

$$V=\sqrt{\frac{E_{max.}\,g\times2}{Rd\,l_h\,\Delta_r}} \quad \text{...............(40)}$$

or $V=1.414\sqrt{\dfrac{1960\times10^8\times980}{200\times150\times450\times2.6}}$

$=1.046\times10^3\text{cm/sec}=10.5\text{m/sec}$.

This may be the maximum additional velocity of the mass of rock if there be no leakage of gases through cracks and a bore hole.

5-3. Functions of stemming

Functions of stemming in blasting of rock may divided into four phases.

(1) Stemming makes detonation of an explosive charge more complete. Stronger is the confinement, the more complete becomes the detonation of an explosive charge and the value of detonation pressure p_D approaches to the theoretical value.

(2) Stemming increases the effective length of shock wave produced by detonation. The effective length L of a shock wave may depends not only on an absolute value of detonation pressure p_D but also on the duration of high pressure which is governed also by the intensity of "back pressure" which may be higher with stronger stemming.

(3) Stemming increases initial pressure of gases at the moment of the beginning of work done against rock. Before a shock wave produces slabs which reach the position of an explosive charge the high pressure gases cannot perform effective work of expansion against rock. It takes time

$$t_s=\frac{2d}{U} \text{.................................(41)}$$

for a shock wave to produce full slabs where d=burden, U=velocity of a shock wave. During this time t_s the pressure of gases may be reduced to a lower value p_b and the amount of pressure reduction (p_D-p_b) depends on the stemming, that is, total weight of stemming and the friction between the wall of a bore hole and stemming. The latter may be described by the following equations for stemming:

$P=Ce^{KL}$...(42) (Y. Shimomura[10])

where P=resistance due to stemming

C=constant

e=base of natural logarithm

K=constant

L=length of stemming

or $P=P_0 e^{\frac{4K\mu L}{d}}$ ......(43) (J. Taylor[11])

P_0=constant.

μ=friction coefficient

d=diameter of bore hole

At about time t_s the slabs formed by a shock wave are in a movable condition and the expansion of gases with an initial pressure p_b, performs work of additional acceleration of rock fragmentation.

(4) Usually at an earlier stage of expansion of gases stemming may be already all blown out of a bore hole. However there may be small contribution from the stemming to the increase of work done by expansion of gases.

The important functions of stemming in blasting may be (1), (2), (3) and not (4). The stages (1), (2) and (3) may be realized within an extremely short period of time, for example, within about half millisecond as is shown by the following relation.

$$t=\frac{2\times 1.5\text{meter}}{5740\text{meter/sec}}=0.000522 \text{ sec. or}$$

0.5millisecond.

Because of this extremely short period of function even a small amount of stemming may be extraordinarily effective in blasting.

10) Y. Shimomura: Journal of the Mining Institute of Japan. 1942, Nov. p. 693.

11) J. Taylor: Mining Engineer, Vol. 96, 1938~1939.

§ 6. Simultaneous blasts and selection of spacing S_C between two bore holes

In Fig. 4. spacing S_C between two cylindrical charges C_1 and C_2 is defined by:

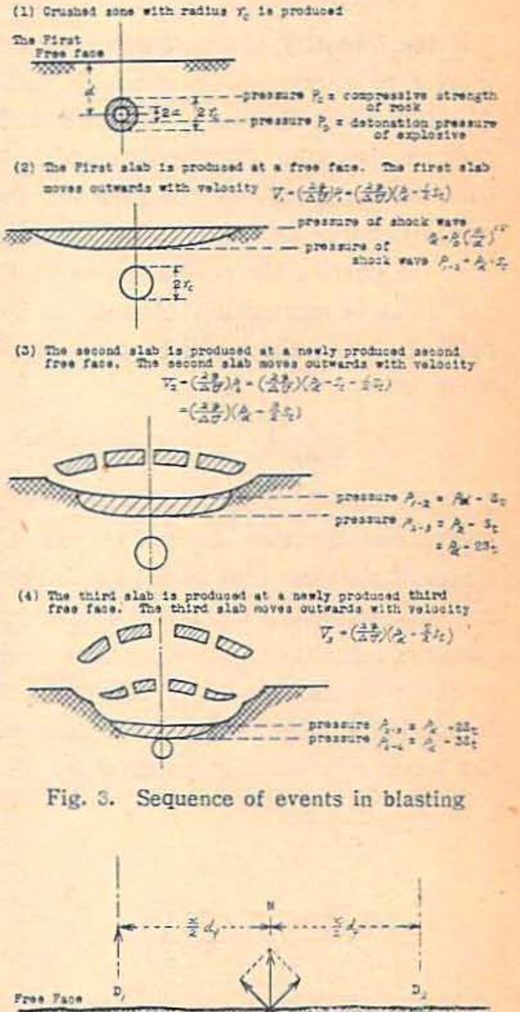

Fig. 3. Sequence of events in blasting

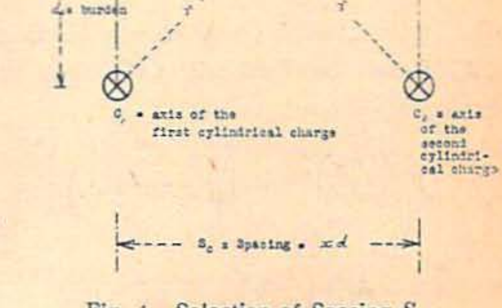

Fig. 4. Selection of Spacing S_C

$$S_0 = xd_f \quad \cdots\cdots(44)$$

where d_f=burden of an individual charge. The problem is to find the numerical value of reduced spacing

$$x = S_0/d_f \quad \cdots\cdots(45)$$

If the intensity of shock wave perpendicular to a free face at a middle point M between two charges is the same with that at the point D_1, (or D_2) which is just above the axis of the first (or the second) charge then it may be assumed that the part of the solid between D_1 C_1 and D_2 C_2 be successfully blasted. At the point D_1 or D_2 the intensity of a shock wave is:

$$p_d = p_D\left(\frac{a}{d_f}\right)^n \quad \cdots\cdots(46)$$

At the middle point M the two shock wave from the two charges C_1 and C_2 arrive at the same time when both charges are simultaneously detonated by use of the same lengths of detonating fuses from a single initiator. Then the outward component of the resultant intensity of the shock wave at the point M is:

$$2p_D\left(\frac{a}{r}\right)^n\left(\frac{d_f}{r}\right)^2$$

$$=2p_D\left(\frac{a}{\sqrt{d_f^2+\left(\frac{x}{2}d_f\right)^2}}\right)^n \cdot \frac{d_f^2}{d_f^2+\left(\frac{x}{2}d_f\right)^2} \quad \cdots\cdots(47)$$

For a successful simultaneous blasting (47) should be equal with (46), then we have:

$$p_D\left(\frac{a}{d_f}\right)^n = 2p_D\left(\frac{a}{\sqrt{d_f^2+\left(\frac{x}{2}d_f\right)^2}}\right)^n \frac{d_f^2}{d_f^2+\left(\frac{x}{2}d_f\right)^2} \quad \cdots\cdots(48)$$

or $$1=2\left(\frac{1}{\sqrt{1+\frac{x^2}{4}}}\right)^n \frac{1}{1+\frac{x^2}{4}} \quad \cdots(49)$$

For an assumed value of n=1.5 we have:

$$\left(1+\frac{x^2}{4}\right)^{\frac{n}{2}+1}=2$$

or $$x=2\sqrt{2^{\frac{2}{n+2}}-1}$$

$$x=1.39 \doteqdot 1.4 \quad \cdots\cdots(50)$$

or $$S_e = 1.4d_f \quad \cdots\cdots(51)$$

The formula (51) may be used for the selection of spacing S_e on condition that an individual charge has a weight and a burden just equal to produce a full crater respectively.

§ 7. Blasting with two free faces.

7-1. Selection of two burdens

In Fig. 5. (1) a cylindrical charge with its axis C perpendicular to the plane of the paper has two burdens d_1 and d_2 to be blasted. If we select d_1 so as the charge with its radius "a" can produce a full crater against the first free face ED then the intensity of a reflected tension wave at C may be assumed to be the same with the tensile strength of rock S_t. If we select the second burden d_2 so as:

$$d_2 = 2d_1 \quad \cdots\cdots(52)$$

then the reflected tension wave with regard to the second free face EF has an intensity equal to S_t at the second free face, that is, at F, therefore, fracture due to tension does not occur.

Now the distance of propagation r' of a shock wave at which the intensity is reduced to a half of the tensile strength of rock is found as follows.

$$\frac{1}{2}S_t = p_D\left(\frac{a}{r'}\right)^n \quad \cdots\cdots(53)$$

while we have:

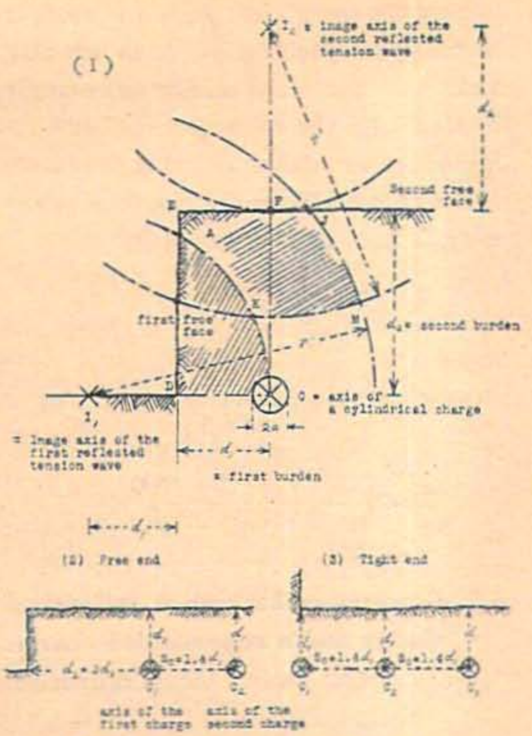

Fig. 5. Blasting with two free faces

$$S_t = p_D\left(\frac{a}{2d_1}\right)^n = p_D\left(\frac{a}{d_2}\right)^n \quad \text{......(54)}$$

From the two equations (53) and (54) we find:

$$\frac{r'}{d_2} = \frac{r'}{2d_1} = 1.58 \quad \text{.....................(55)}$$

In Fig. 5. each of the wave front MJ and MK has an intensity equal to $\frac{1}{2}S_t$ then a part of rock $MJEAK$ may be assumed to be broken by the cooperation of two waves each of which has an intensity $\frac{1}{2}S_t$ although the degree of fragmentation may be different from that in a full crater CAD.

7-2. Difference between free and tight end.

As is illustrated in Fig. 5. (2) and (3) loading factor, F that is weight of an explosive charge per ton of rock broken, is quite different between free end and tight one. Burden d_1 has been selected to be a full crater burden. Then the ratio of loading factor for both cases are.

$$\frac{F \text{ free}}{F \text{ tight}} = \frac{\dfrac{W}{2d_1 \times d_1}}{\dfrac{2W}{1.4d_1 \times d_1}}$$

$$= \frac{1.4}{4} = 0.35 \quad \text{.........................(56)}$$

where W=weight of charge per hole.

§ 8. A cylindrical charge of a limited length

It has been assumed so far in the present paper that a cylindrical charge has an infinite length. Practically two end effects must be taken into consideration. They are (1) the selection of sub-drilling l_s in Fig. 1. and (2) selection of a length of stemming l_n near a mouth of a bore hole supposing the stemming occupies the remaining part of a bore hole left unloaded by a continuous charge whose total length is l_c.

8-1. Selection of sub-drilling

The total length l_c of a long cylindrical charge may be divided into three parts, that is, l_1, l_2 and l_3 as are illustrated in Fig. 1. The parts l_1 and l_3 which constitute two ends of a cylindrical charge may be assumed to work as concentrated charges rather than cylindrical ones in a sense that the shock waves produced by parts l_1 and l_3 are propagated as spherical waves into three dimensions while a shock wave produced by the part l_2 is propagated as a cylindrical wave into two dimensions.

The upper limit of the sub-drilling may be found if we assume that the part l_3 with its center of gravity on the grade line ED must be of a sufficient quantity to blast a toe burden d_1. If we describe a corresponding radius of a spherical charge by a_s whose total volume is equal to the volume represented by l_3 then we have the following relation.

$$\frac{4}{3}\pi a_s^3=\pi l_3 a^2 \text{ ....(57)}$$

or

$$a_s=\left(\frac{3l_3}{4}\right)^{\frac{1}{3}} a^{\frac{2}{3}} \text{ ....(58)}$$

As l_s=sub-drilling$=\frac{1}{2}l_3$

$$a_s=\left(\frac{3}{2}l_s\right)^{\frac{1}{3}} a^{\frac{2}{3}} \text{ ....(59)}$$

As a cylindrical wave produced by the main part l_2 must be strong enough to produce a full fracture along a main free face, we have:

$$d_f=d_1,\ S_t=p_D\left(\frac{a}{2d_f}\right)^{1.5} \text{ ....(60)}$$

While for a concentrated charge l_3 we have:

$$S_t=p_D\left(\frac{a_s}{2d_f}\right)^2 \text{ ....(61)}$$

From the equations (59), (60) and (61) we have:

$$l_s=\frac{4}{3}\left(\frac{S_t}{p_D}\right)^{\frac{1}{6}}d_f \text{ ....(62)}$$

or

$$l_s=\frac{4}{3}\left(\frac{55}{160\times10^3}\right)^{\frac{1}{6}}d_f=0.35d_f \text{ (63)}$$

8-2. Selection of the length of stemming l_n.

The length of stemming may be derived if we assume that the part l_1 acts as a spherical charge. To blast a burden d_f sucessfully against the first free face AF the length l_1 should be:

$$l_1=l_3=2l_s=\frac{8}{3}\left(\frac{S_t}{p_D}\right)^{\frac{1}{6}}d_f \text{ ....(64)}$$

If we choose the burden $\left(l_n+\frac{1}{2}l_1\right)$ against the second free face AB so as it becomes the double of d_f then we may assume the part $C_1\ F\ A\ B$ be successfully blasted[12] by the co-operation of two reflected tension waves from two free faces AF and AB. Then the length of stemming l_n may be represented by:

$$l_n=2d_f-\frac{1}{2}l_1$$

or

$$l_n=2\left\{1-\frac{2}{3}\left(\frac{S_t}{p_D}\right)^{\frac{1}{6}}\right\}d_f \text{ ....(65)}$$

or

$$l_n=2\left\{1-\frac{2}{3}\left(\frac{55}{160\times10^3}\right)^{\frac{1}{6}}\right\}d_f$$
$$=1.65d_f \text{ ....(66)}$$

§ 9. Comparison between a cylindrical charge and a concentrated charge.

9-1. Loading factor for a cylindrical charge, Fig. 6-(1)

For blasting with cylindrical charges as is illustrated in Fig. 6-(1) the volume of rock broken V per bore hole is as follows:

$$V=H\,S_e\,d_f \text{ ....(67)}$$

H=height of face

where

S_e=spacing between bore holes

d_f=burden for complete fracture

By the equation (51) $S_e=1.4d_f$, and by (63) and (66)

$$H=l_c-l_s+l_n=l_c-0.35d_f+1.65d_f$$
$$=l_c+1.3d_f \text{ ....(68)}$$

For complete fracture over a burden d_f the approximate equation (13) $S_t=p_D\left(\frac{a}{2d_f}\right)^{1.5}$ holds from which we have:

12) Kumao Hino: Theory and practice of coyote blasting; This Journal, 17, 156-171 (1956)

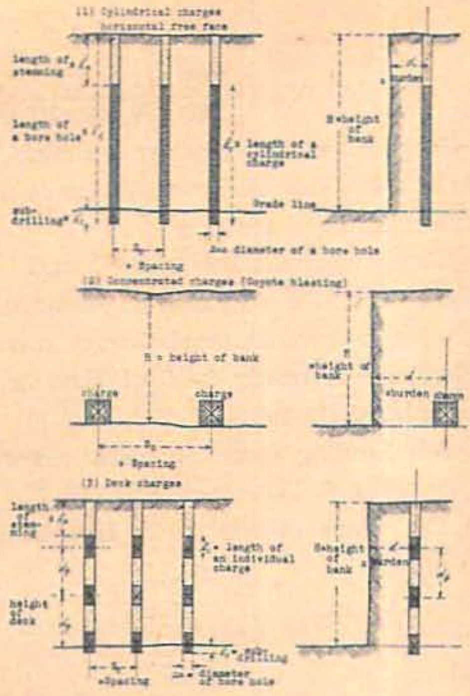

Fig. 6. Comparison between a cylindrical charge and a concentrated charge.

$$d_f=\frac{a}{2}\left(\frac{p_D}{S_t}\right)^{\frac{1}{1.5}} \quad \text{.................}(70)$$

Let us assume that a total length of a cylindrical charge l_c is much longer than 1.3 d_f in the equation (66).

Then we have:

$$V=1.4l_c\left(\frac{a}{2}\right)^2\left(\frac{p_D}{S_t}\right)^{\frac{2}{1.5}} \quad \text{.........}(71)$$

Weight of charge W per hole is:

$$W=\pi a^2 l_c \Delta_e \quad \text{........................}(72)$$

where Δ_e=loading density of an explosive charge.

If we denote the density of rock by Δ_r we find the loading factor as follows: loading factor

$$f=\frac{W}{V\Delta_r}=2.86\pi\left(\frac{\Delta_e}{\Delta_r}\right)\left(\frac{S_t}{p_D}\right)^{1.33} \quad \text{...........................}(73)$$

If we use W grams of explosive charge for $V\Delta_r$ tons of rock broken then the loading factor (grams of explosive consumed to break one ton of rock) F is expressed by the following equation.

$$F=2.86\times10^6\pi\left(\frac{\Delta_e}{\Delta_r}\right)\left(\frac{S_t}{p_D}\right)^{1.33} \quad \text{..........................}(74)$$

$$\text{or } F=2.86\times10^6\times3.14\times\left(\frac{1.45}{2.63}\right)\left(\frac{55}{160\times10^3}\right)^{1.33}$$

=122 grams of Ammon Gelatin (Shin Kiri Dynamite) for one ton of limestone broken.

If we take cubic meters of rock to be broken instead of tons of rock then the loading factor F' is represented by the following formula:

$$F'=2.86\times10^6\pi\,\Delta_e\left(\frac{S_t}{p_D}\right)^{1.33} \quad \text{...}(75)$$

or $F'=122\times2.63=321$ grams of explosives for one cubic meters of rock to be broken. The formulas (74) or (75) may be used to estimate "effective tensile strength S_t of rock" because we know numerical values of "loading factors" through experience for a given pair of an explosive and rock.

9-2. Loading factor for a concentrated charge (coyote blasting) Fig. 6-(2)

The ideally concentrated charge is realized in the so-called "coyote blasting".[12] The volume of rock broken per chamber is:

$$V=H S_c d_f \quad \text{..........................}(76)$$

If we assume that the concentrated charge be represented by a sphere with radius "a" the weight of explosive per chamber is:

$$W=\frac{4}{3}\pi a^3 \Delta_e \quad \text{....................}(77)$$

By the same reasoning as is described in § 6. the spacing S_c is determined by the following relations.[13]

$$S_e = x\, d_f \quad \text{.............................. (78)}$$

$$\left(1+\frac{x^2}{4}\right)^{\frac{n}{2}+1} = 2 \quad \text{.................. (79)}$$

For a spherical charge

$$n=2, \quad x=1.3 \text{ or } S_e = 1.3 d_f \quad \text{...... (80)}$$

The height of bank may be[12] represented by the following relation.

$$H = 2d_f \quad \text{.................................. (81)}$$

For a burden to be completely raised the following relation must be realized.[14]

$$S_t = p_D\left(\frac{a}{2d_f}\right)^{\frac{4}{3}} \quad \text{...................... (82)}$$

or

$$d_f = \frac{a}{2}\left(\frac{p_D}{S_t}\right)^{\frac{3}{4}} \quad \text{.................. (83)}$$

Combining the equations (76), (77), (80), (81) with (83) we find the loading factor F as follows:

$$F = 4.05\times 10^6 \pi \left(\frac{\Delta_e}{\Delta_r}\right)\left(\frac{S_t}{p_D}\right)^{1.5}$$

$$\frac{\text{gram explosive}}{\text{ton of rock}} \quad \text{............ (84)}$$

$$\text{or} \quad F = 4.05\times 10^6 \pi \left(\frac{1.45}{2.63}\right)\left(\frac{55}{160\times 10^3}\right)^{1.5}$$

$=44$ grams explosive per ton of rock

or $F' = 44\times 2.63 = 116$ grams explosive per cubic meters of rock.

The comparison between 9-1 and 9-2 shows that the consumption of an explosive in a concentrated charge (coyote blasting) is nearly half of that in a cylindrical charge.

9-3. Loading factor for deck charges Fig. 6. (3)

The volume of rock to be broken per bore hole is:

$$V = S_e d\left(l_s + N_d\, d_y + \frac{1}{2} l_e + l_n\right) \quad \text{... (85)}$$

where

- N_d = number of deck
- d_y = height of deck or vertical distance between two decks
- S_e = spacing between bore holes
- l_e = length of a short cylindrical charge which may be assume to act as a concentrated charge with radius "a_s"
- l_s = sub-drilling
- l_n = length of stemming near the mouth of a bore hole.

d_y may be assumed to be the same with spacing S_e to secure the complete fragmentation.

$$d_y = S_e = 1.3 d_f \quad \text{........................ (86)}$$

$$\frac{4}{3}\pi a_s^3 = \pi l_e a^2 \quad \text{or}$$

$$a_s = \left(\frac{3}{4} l_e\right)^{\frac{1}{3}} a^{\frac{2}{3}} \quad \text{..................... (87)}$$

To break a burden d_f completely with an individual charge the following relation must be realized.

---

13) In previous papers[1] and[12] the upward component of resultant shock wave at a free face at a middle point M has been assumed to be $2p_D\left(\frac{a}{r}\right)^n \frac{d}{r}$ while, strictly speaking, it should be $2p_D\left(\frac{a}{r}\right)^n\left(\frac{d}{r}\right)^2$. This difference makes the value of x change from 1.54 into 1.29.

The equation (30) of reference[1] becomes:

$$\left(\frac{R}{d}\right)^2 + 1 = \left(\frac{p_D}{S_t}\right)^{\frac{2}{2+n}}\left(\frac{a}{d}\right)^{\frac{2n}{n+2}}$$

and the equation (5) of reference[14] and the equation (1) of reference[12] become.

$$\frac{d_f}{a} = D_f = \left(\frac{p_D}{S_t}\right)^{\frac{1}{n}}\left\{\left(\frac{R}{d_f}\right)^2 + 1\right\}^{-\left(\frac{2+n}{2n}\right)}$$

That is, the coefficients are slightly changed from $(1+n)$ into $(2+n)$

14) Kumao Hino; Concentrated type of no-cut round of blasting. This Journal, **16**, No. 3, 173. (1955)

$$d_f=\frac{a_s}{2}\left(\frac{p_D}{S_t}\right)^{\frac{1}{2}} \qquad (88)$$

$$l_n+\frac{1}{2}l_e=2d_f \qquad (89)$$

Total weight of charges W per bore hole is as follows:

$$W=N_d\,\pi\,a^2\,l_e\,\Delta_e \qquad (90)$$

For a large number of N_d we may neglect $\left(\frac{1}{2}l_e+2d_f\right)$ against $1.3d_f\,N_d$ then the combination of the equations (85), (86), (87), (88), (89) with (90) gives the loading factor F.

$$F=6.31\times10^6\pi\left(\frac{\Delta_e}{\Delta_r}\right)\left(\frac{S_t}{p_D}\right)^{1.5}\ \frac{\text{gram explosive}}{\text{ton of rock}} \qquad (91)$$

or $F=6.31\times10^6\pi\left(\frac{1.45}{2.63}\right)\left(\frac{55}{160\times10^3}\right)^{1.5}$

$=69\ \frac{\text{gram explosive}}{\text{ton of rock}}$

or $F'=69\times2.63=181\ \frac{\text{gram explosive}}{\text{cubic meters of rock}}$

The result shows that the deck charges consume less explosive than a cylindrical charge while they consume more explosive than coyote blasting.

9-4. Fragmentation in three cases

Fragmentation depends on the absolute size of blasting round, therefore, we must specify the height of a bank H and bit gauge $2a$ which is available.

For example, suppose $H=20$ meter, $2a=25\text{cm}\fallingdotseq10$ inches.

9-4A. Cylindrical charges.

For a cylindrical charge the peak pressure of a shock wave at a free face is as follows:

$$p_d=p_D\left(\frac{a}{d}\right)^{1.5} \qquad (92)$$

while d is determined by

$$S_t=p_D\left(\frac{a}{2d}\right)^{1.5} \qquad (93)$$

combining (92) with (93) we find:

$$p_d=2^{1.5}S_t \qquad (94)$$

or $p_d=2^{1.5}\times55=2.8\times55=154\text{Kg/cm}^2$.

The number of slabs N may be estimated by the following formula.[1)]

$$N=\frac{p_d}{S_t} \qquad (95)$$

or

$$N=2^n=2^{1.5}=2.8 \qquad (95)'$$

The thickness of a slab l may be described as follows:

$$l=\frac{d}{N} \qquad (96)$$

while from the equation (93):

$$\frac{d}{a}=\frac{1}{2}\left(\frac{p_D}{S_t}\right)^{\frac{1}{1.5}} \qquad (94)$$

or

$$d=\frac{a}{2}\left(\frac{p_D}{S_t}\right)^{\frac{1}{1.5}} \qquad (95)$$

or $d=\frac{25}{2\times2}\left(\frac{160\times10^3}{55}\right)^{\frac{1}{1.5}}=\frac{25}{4}\times170.2=$ 1063cm.

Then

$$l=\frac{a}{2^{n+1}}\left(\frac{p_D}{S_t}\right)^{\frac{1}{n}} \qquad (96)$$

or $l=\frac{1063}{2.8}=380\text{cm}$.

9-4B. Concentrated charge

For a coyote blasting the peak pressure of a shock wave at a free face is as follows:

$$p_d=p_D\left(\frac{a}{d}\right)^2 \qquad (97)$$

while d is determined by:

$$S_t=p_D\left(\frac{a}{2d}\right)^2 \qquad (98)$$

combined (97) with (98) we have:

$$p_d=2^nS_t=2^2S_t \qquad (99)$$

or $p_d=4S_t$

$p_d=4\times 55\text{kg/cm}^2=220\text{kg/cm}^2$

The number of slabs N is:

$$N=\frac{p_d}{S_t}=2^n=4 \quad \text{..................(100)}$$

As the height of a bank H is double of the burden d the thickness of a slab l is as follows:

$$l=\frac{d}{N}=\frac{H}{2N} \quad \text{.....................(101)}$$

or $l=\dfrac{2000\text{cm}}{2\times 4}=250\text{cm}.$

9-4C. Deck charges

For a deck charge the peak pressure of a shock wave at a free face is as follows:

$$p_d=p_D\left(\frac{p_t}{d}\right)^2 \quad \text{.....................(102)}$$

while

$$a_s=\left(\frac{3}{4}l_e\right)^{\frac{1}{3}}a^{\frac{2}{3}} \quad \text{.....................(87)}$$

Let us assume that the length of an individual charge l_e has been taken so as:

$$l_e=2a \quad \text{.................................(103)}$$

The condition (103) realizes the assumption of a concentrated charge, then, we have:

$$a_s=1.15a \quad \text{..............................(104)}$$

The condition for a full fragmentation is:

$$S_t=p_D\left(\frac{a_s}{d}\right)^2=p_D\left(\frac{1.15a}{d}\right)^2 \text{...(105)}$$

From (102) and (105) we have:

$$p_d=4S_t \quad \text{..............................(106)}$$

The number of slabs N is:

$$N=\frac{p_d}{S_t}=2^n=2^2=4 \quad \text{............(107)}$$

The thickness l of a slab is:

$$l=\frac{d}{N}=\frac{\frac{a_s}{2}\left(\frac{p_D}{S_t}\right)^{\frac{1}{2}}}{2^2}=\frac{a_s}{8}$$

$$\left(\frac{p_D}{S_t}\right)^{\frac{1}{2}}=\frac{1.15a}{8}\left(\frac{p_D}{S_t}\right)^{\frac{1}{2}} \text{...(108)}$$

or $l=1.15\times\dfrac{25}{2\times 8}\left(\dfrac{160\times 10^3}{55}\right)^{\frac{1}{2}}=97\text{cm}.$

The burden d is:

$$d=\frac{1.15a}{2}\left(\frac{p_D}{S_t}\right)^{\frac{1}{2}}=388\text{cm}\text{......(109)}$$

The results described above show that deck charges give the best fragmentation (97cm) while coyote blast gives rather larger fragments (250cm) and cylindrical charges give the largest fragments (380 cm).

9-4D. Number of bore holes in cylindrical charges and deck charges.

It must be noted that burdens are 388 cm for deck charges, 1000cm for coyote blasting and 1063cm for cylindrical charges respectively.

It means that number of bore holes per cubic meters of rock to be broken is drastically increased when the magnitude of burdens is decreased as is illustrated in Fig. 7.

The number of bore holes B per unit area of a horizontal face is for cylindrical charges as follows:

$$B'=\frac{1}{S_e d}=\frac{1}{1.4d^2}=\frac{1}{1.4}\left\{\frac{2}{a}\left(\frac{S_t}{p_D}\right)^{\frac{1}{1.5}}\right\}^2=\frac{2.86}{a^2}\left(\frac{S_t}{p_D}\right)^{1.33}$$

$$\text{....................................(110)}$$

or $B'=\dfrac{2.86\times 4}{25^2}\left(\dfrac{55}{160\times 10^3}\right)^{1.33}=4.45\times 10^{-7}$ per square cm.

or $B=10^6\,B'=0.45$ bore holes for 100m^2.

Therefore one bore hole is needed for $\dfrac{100}{0.45}=220\text{m}^2$.

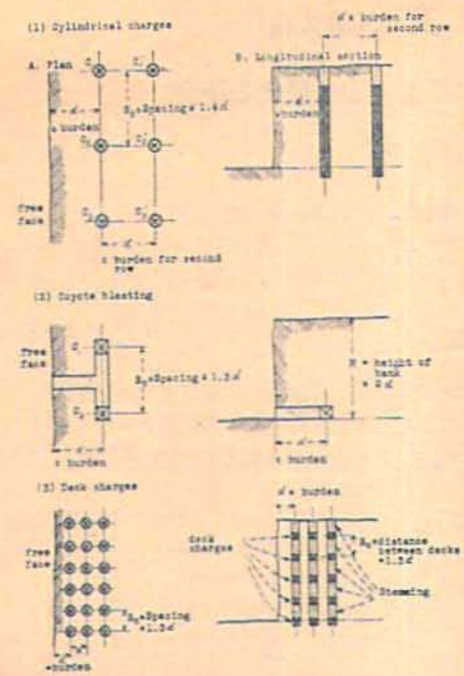

Fig. 7. Blasting patterns for cylindrical charges, coyote blasting and deck charges.

In the case of deck charges

$$B'=\frac{1}{S_e d}=\frac{1}{1.3d^2}=\frac{1}{1.3}\left\{\frac{2}{1.15a}\left(\frac{S_t}{p_D}\right)^{\frac{1}{2}}\right\}^2=\frac{2.33}{a^2}\left(\frac{S_t}{p_D}\right)\quad\ldots(111)$$

or $B'=\frac{2.33\times4}{25^2}\times\frac{55}{160\times10^3}=51.4\times10^{-7}$ per square cm.

or $B=10^6$ $B'=5.1$ hore holes for 100m^2. Therefore one bore hole is needed for $\frac{100}{5.1}=19.6\text{m}^2$. Roughly speaking in the case of deck charges we need $\frac{220}{19.6}=11.2$ times more bore holes than in the case of cylindrical charges.

Table 2. summarizes the results of discussion described above.

(Continued)

Table 2. Comparison between cylindrical charges and concentrated charges.

	Types of blasting	Loading factor F=gram explosive	Height of bank H=20m. diameter of bore hole $2a$=25cm.			
		cubic meter rock	Burden $d$cm	Spacing $S_e=\times d$	Thickness of a slab $l$cm	Number of bore holes for 1000m²
(1)	Cylindrical charge	321g/m³	1063cm	1.4$d$cm	380	4.5
(2)	Concentrated charge (coyote)	116g/m³	1000cm	1.3$d$cm	250	—
(3)	Deck charge	181g/m³	388cm	1.3$d$cm	97	51

Note: detonation pressure of an explosive $p_D=160\times10^3$ kg/cm²
tensile strength of rock $S_t=55$kg/cm²
loading density of explosive $\Delta_e=1.45$g/cm³
density of rock $\Delta_r=2.63$g/cm³