Research paper

A new estimating method of the Hugoniot

Shiro Kubota*†, Tei Saburi*, Yuji Ogata*, and Kunihito Nagayama**

*Research Core for Explosion Safety, Research Institute Science for safety and Sustainability, National Institute of Advanced Industrial Science and Technology, 16-1 Onogawa, Tsukuba, Ibaraki, 305-8569 JAPAN
TEL +81-29-861-8700 FAX +81-29-861-8700
†Corresponding address : kubota.46@aist.go.jp

**Faculty of Engineering, Kyushu University, Fukuoka, 819-0395, JAPAN
E-mail : nagayama@aero.kyushu-u.ac.jp

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Abstract

The new idea for estimating Hugoniot of the materials based on the formulation of a non steady flow of the shock wave propagation has been proposed. In general, though the Hugoniot measurement employs the shock wave with perfectly square waveform, a proposed method can be applied even for the decayed shock wave. This method can determine the relationship of the Hugoniot on particle velocity-shock velocity, and the local slope of the Hugoniot by using two profiles of the particle velocity.

Keywords : Hugoniot, decayed shock wave, particle velocity, slope of the Hugoniot

1. Introduction

Hugoniot measurement for condensed matter, in general, employs the flat plate impact method[1),2)] or a shock attenuation system with an explosive lens[3),4)] to realize the ideal one-dimensional condition. The advantages of using flat plate impact system are that a shock wave with a perfectly square waveform can be generated and the duration can be controlled by changing the length of the projectile. In the case of a shock wave attenuation system with an explosive lens, the system can produces a shock wave with a roughly square waveform. In both methods, by realizing sustained shock loading into the sample material, a jump condition is retained. As a result, the basic theory can be applied for data analysis. On the other hand, these measuring methods require large-scale equipment or a large amount of explosive, as well as careful maintenance. If the Hugoniot estimation method allows the shock wave to decay behind the shock front, the simpler equipment or a shock load generated by a smaller amount of explosive can be used for estimating the Hugoniot of materials.

A new idea for estimating the material properties under shock loading is proposed in this paper. First a method of estimating one point on the Hugoniot is investigated using those recorded results of numerical simulations, which were performed to extract the profiles corresponding to by a gauge in Hugoniot measurement. Second the exact formulation for the shock wave decay process is applied to estimate the local slope of the Hugoniot using less information than that required for conventional method.

2. Exact formulation of shock wave decay process, concept of the new idea for estimation of the Hugoniot, and its local slope

2.1 Exact formulation of shock wave decay process

The formulation of a non steady flow in the case of shock wave propagation can be expressed as

$$\frac{\delta u_f}{\delta t} = f_1(u_f, u_s, B) \times \left[\frac{\alpha(u_s - u_f)}{R(t)} \times f_2(u_f, u_s, \Gamma, B) + B[2(u_s - u_f) - \Gamma u_f] \times \left(\frac{\partial u}{\partial r} \right)_f \right] \quad (1)$$

$$f_1(u_f, u_s, B) = -\frac{u_f u_s}{(u_s - u_f B)(2u_s + u_f B)} \quad (2)$$

$$f_2(u_f, u_s, \Gamma, B) = u_s + u_f B - \frac{\Gamma u_f^2}{u_s - u_f} B \quad (3)$$

where, u_f and u_s are the particle velocity, and the propagation velocity of the shock wave, respectively. The symbol δ corresponds to the variation along the shock front and B is the derivative of the u_s with respect to u_f, i.e., du_s/du_f. Γ is the Gruneisen coefficient. The symbol α takes a value of 0 for a slab geometry,1for cylindrical geometry, and 2 for a spherical geometry. $R(t)$ is the coordinate of the shock front at time t. The formulation for the non steady flow due to shock wave propagation was previously derived by one of the authors on the basis of the conservation law [5),6)], and is rewritten in the above form to estimate the properties of materials under shock loading in this study.

In general, the measurement of particle velocity at the free surface or that of the pressure history using a gauge embedded in the sample has been employed to determine the Hugoniot. We assume that the profile of the particle velocity could be obtained from the measurement of the free surface velocity, and that the average velocity of the shock wave could be also obtained from the length between two points and the arrival time of shock wave at each point.

To use the profile of the particle velocity to estimate the Hugoniot using Eq. (1), the following transformation was employed :

$$\left(\frac{\partial u}{\partial r}\right)_f = \frac{1}{u_s - u_f}\left[\frac{\delta u_f}{\delta t} - \frac{Du_f}{Dt}\right] \tag{4}$$

where Du_f/Dt corresponds to the local slope of the profile of the particle velocity.

2.2 Concept of the new method for estimation of the Hugoniot

The known variables, which are measured by experiments, are $u_{f1}(t)$, $u_{f2}(t)$, and $\overline{u}_s$. Because $\overline{u}_s$ is the average value estimated by two points and these arrival time, the both of (u_{f1}, $\overline{u}_s$) and (u_{f2}, $\overline{u}_s$) may not be on the Hugoniot. Here, $\overline{u}_s$ is

$$\overline{u}_s = \frac{R(t_1) - R(t_2)}{t_1 - t_2} \tag{5}$$

Therefore to estimate the Hugoniot, we use the average particle velocity $\overline{u}_f$, given by

$$\overline{u}_f = \frac{u_{f1} + u_{f2}}{2} \tag{6}$$

The left hand−side of Eq. (1) is also equivalent to the following average value :

$$\frac{\delta u_f}{\delta t} = \frac{u_{f1} - u_{f2}}{t_1 - t_2} \tag{7}$$

There are still two unknown variables B and Γ in Eq. (1). Since the influence of Γ to the calculation results is less for many condensed media, the empirical relationship between B and Γ is applied herein as follows.

$$\Gamma = 2B - 1 \tag{8}$$

In this paper, this relationship is employed to estimate the local slope of the Hugoniot. Because we have two equations, i.e. Eqs. (1) and (8), the local slope B of the Hugoniot can be obtained by these equations. If a material has a well −known linear relationship between the particle velocity and the shock velocity, the entire Hugoniot can be determined using this method. We investigate the feasibility this concept by performing numerical simulations on materials with a known linear relationship between u_f and u_s to construct the new Hugoniot estimation method.

Table 1 Shock properties of the selected materials ($u_s = A + Bu_f$), Γ; Gruneisen coefficient.

	A ($km{\cdot}s^{-1}$)	B	Γ
PMMA	2.43	1.5785	2.157
Aluminum	5.35	1.35	1.7
Copper	3.958	1.497	2

3. Procedure for investigating for proposed method by numerical simulation

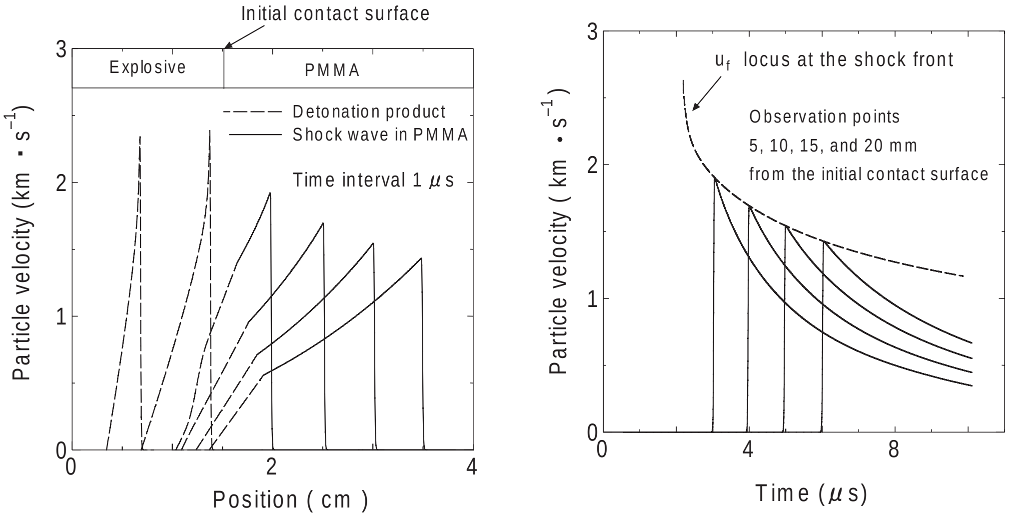
One−dimensional numerical simulations for slab geometry using a Lagrangian code were conducted to prove the proposed method for Hugoniot estimation. A shock wave with a decay waveform behind the shock front was cased by the detonation of a thin layer of Composition C4 explosive, and the detonation wave was simulated using the ignition and growth model[7)]. Because it was only important to generate the decay shock wave in this study, the explanation concerning the explosive is omitted. For the sample materials, PMMA, aluminum, and copper were selected, the shock properties are shown in Table 1[8)]. The Gruneisen equation of state with Hugoniot as a reference line was employed to model the materials.

Figure 1 shows the particle velocity distributions and the profiles of the particle velocity in PMMA obtained by numerical simulation. In this case, the thickness of the explosive was 15 mm and the initial mesh size was 50 μm. The 3 mm thickness of the explosive was also used. The particle velocities $u_f(t)$ at the points 5, 10, and 15 mm from the initial contact surface between the explosive and the material were obtained by numerical simulation. The u_f at the shock front defined by the peak value of the particle velocity distribution in the material were also obtained every time step to reproduce a locus of $\delta u_f/\delta t$. The value of $\overline{u}_s$ can be calculated from Eq. (5) since the shock velocity can be estimated from only its average value between two points in the experiment. The data obtained by the simulation was used to reproduce the experiment.

4. Discussion

4.1 Estimation of Hugoniot

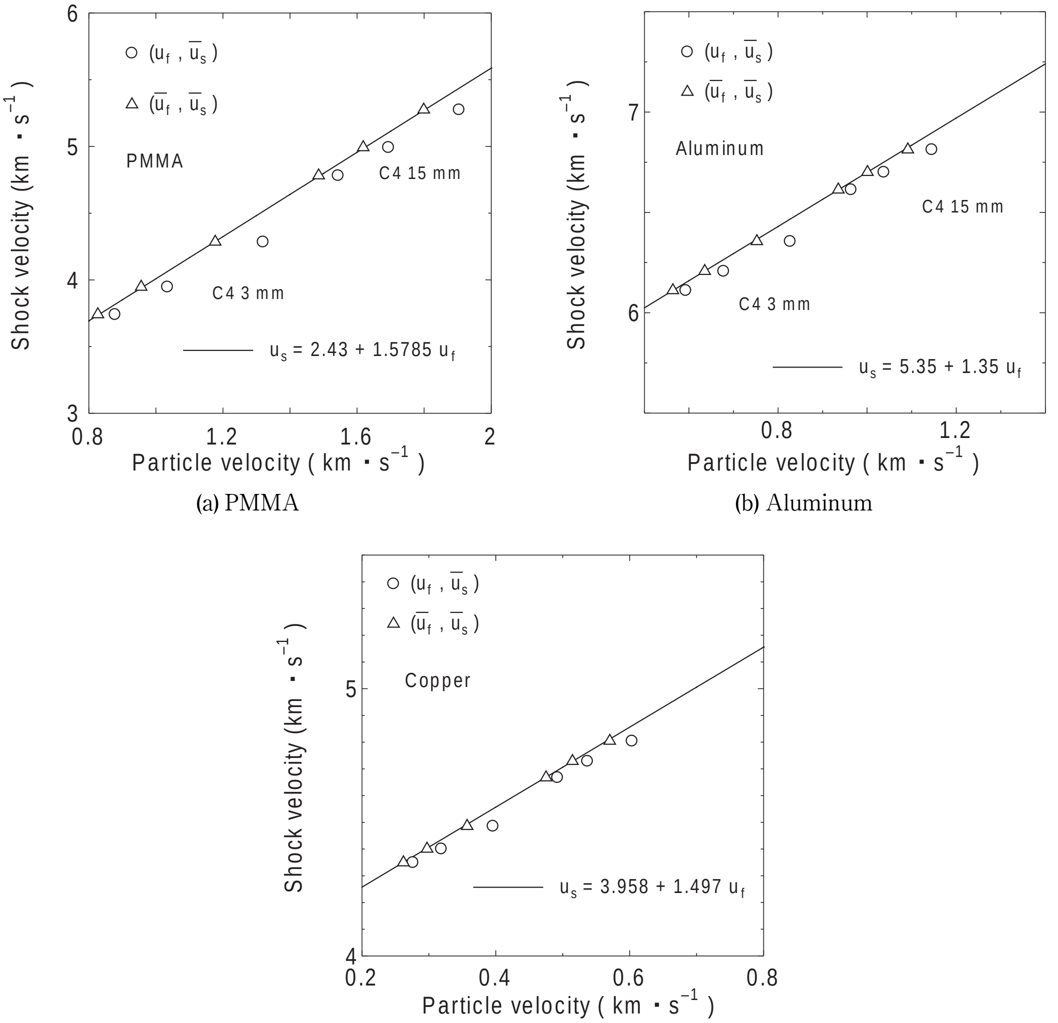
Figure. 2 shows the relationship between the shock velocity and the particle velocity obtained by numerical simulation. It can be found from the figure that the plotted points with coordinates (u_f, $\overline{u}_s$) are removed from the straight line of the Hugoniot. In the case of 3 mm of C4 explosive, the discrepancy between the (u_f, $\overline{u}_s$) and the straight line becomes large together with increase of the particle velocity. This trend can be confirmed for all cases. However, all points of (u_f, $\overline{u}_s$) are the almost on the Hugo-



(a) Particle velocity distributions

(b) Profiles of the particle velocity

Fig. 1 Typical results of the numerical simulations and the propagation process of decay shock wave in PMMA.



(a) PMMA

(b) Aluminum

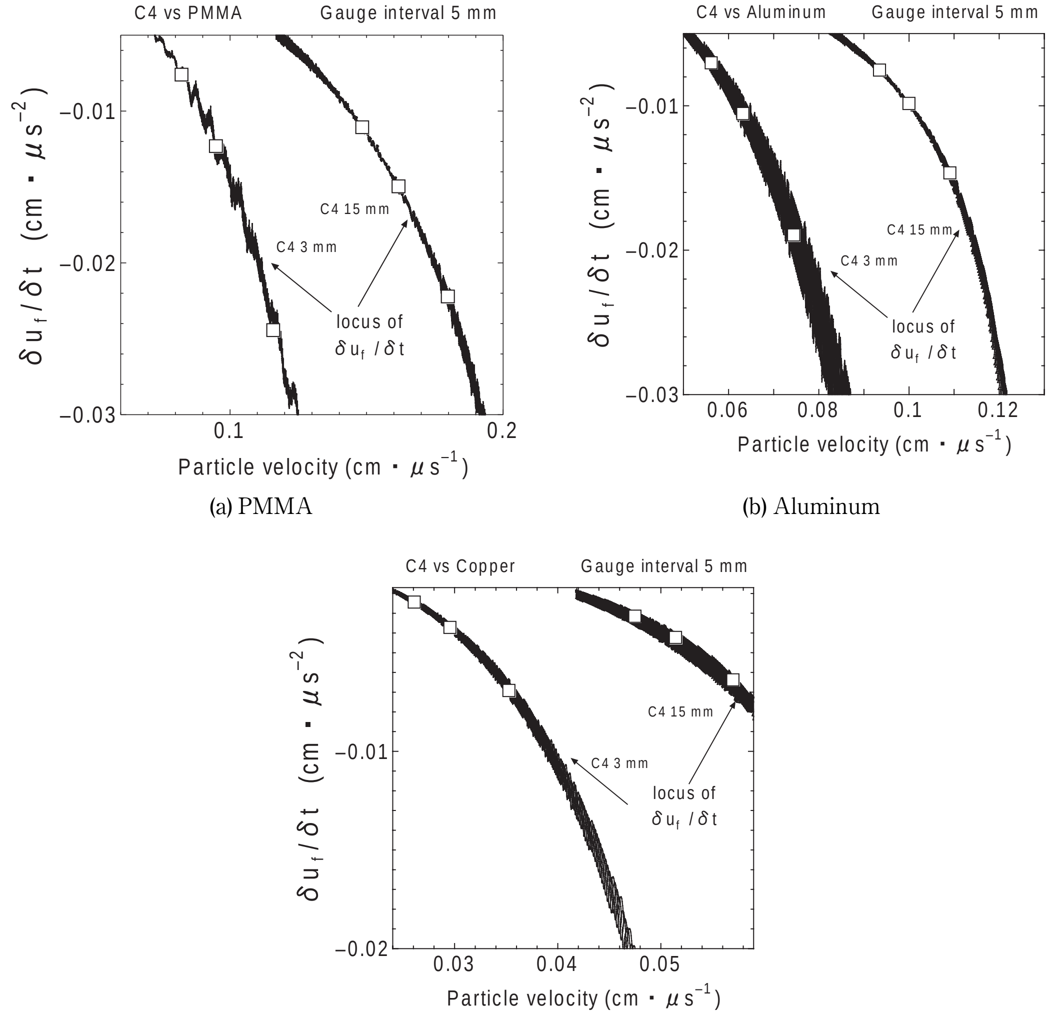
(c) Copper

Fig. 2 The relationship between the shock velocity and the particle velocity obtained by numerical simulation.

niot line. This implies that even in the case of an attenuating shock wave, the Hugoniot with good accuracy can be determined by performing two experiments.

4.2 Estimation of local slope of Hugoniot by new method

We assume that $u_{f1}(t)$, $u_{f2}(t)$, and $\overline{u}_s$ are known. Only these variables are used to estimate the local slope of the Hugoniot. In the previous section, we indicated that the average values of $(\overline{u}_f, \overline{u}_s)$ between two points give a good



(a) PMMA (b) Aluminum

(c) Copper

Fig. 3 The locus of the $\delta u_f/\delta t$ along the shock front as function of the particle velocity. The symbol □ corresponds to two points average values of $\delta u_f/\delta t$ defined by Eq. (7) at $\overline{u}_f$

approximation to the Hugoniot on the shock velocity−particle velocity plane. Therefore, we used these average values of ($\overline{u}_f$, $\overline{u}_s$), which were substituted into Eq. (1) to estimate the local slope of the Hugoniot.

The left hand side of Eq. (1), $\delta u_f/\delta t$, has to be estimated.

The locus of the $\delta u_f/\delta t$ along the shock front as function of the particle velocity obtained by numerical simulation was drawn in Fig. 3. The locus becomes the thick line due to the numerical oscillation. The symbol *square* corresponds to two points average values of $\delta u_f/\delta t$ defined by Eq. (7) at $\overline{u}_f$. Because all points of ($\overline{u}_f$,$\delta u_f/\delta t$) plotted by symbol *square* are located on the locus of $\delta u_f/\delta t$, it is considered that this average value can be applied for this method.

There is one parameter that was not investigated : the slope of the profile of the particle velocity. A conceptual diagram of our method is shown in Fig. 4. Because the average shock velocity is only available, our method is applied the value between t_1 and t_2. The unknown parameter in this diagram are the slopes of particle velocity at $\overline{u}_f$, and $\varDelta t_1$. This diagram was made using the calculation results. The slopes that must be predicted is between Du_{f1}/Dt and Du_{f2}/Dt, and will be expressed by $(Du_f/Dt)^*$ in this paper. It is important to investigate the relationship between the

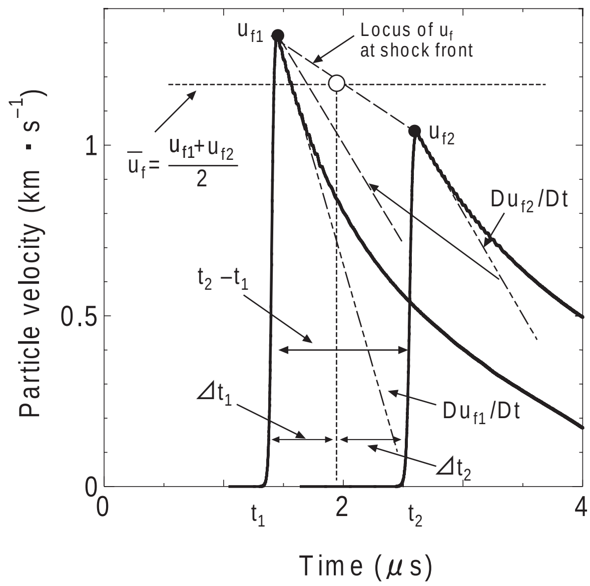


Fig. 4 Conceptual diagram of our method, Two profiles of particle velocity are drawn using the calculation results in the case of PMMA vs 3 mm C4.

arrival times of the shock wave and the slope of the particle velocity to consider the position of the predicted value, and the relationship between the particle velocity and the

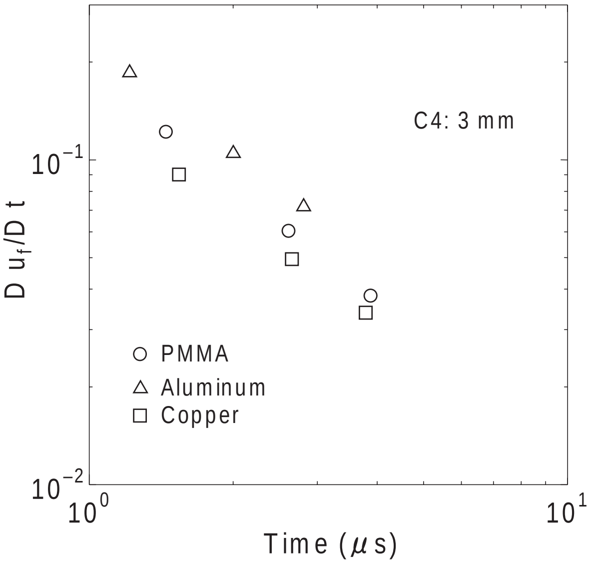


Fig. 5 The relationship between the arrival times of the shock wave and the slope of the particle velocity.

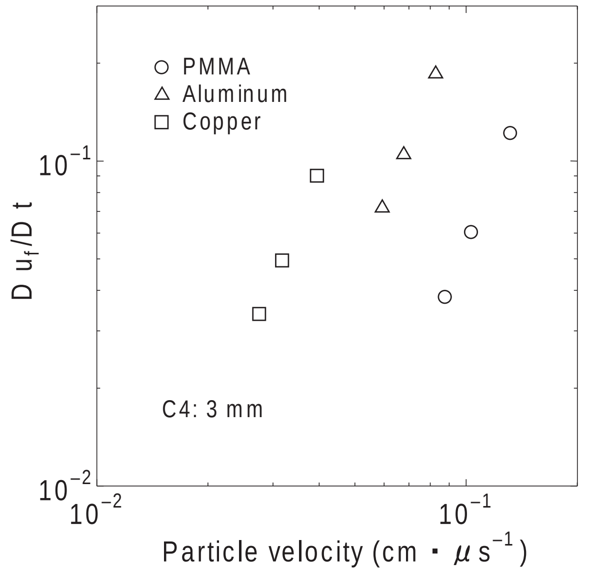


Fig. 6 The relationship between the particle velocity and the slope of the particle velocity.

slope of the particle velocity is also important. These relationships are shown in Figs 5 and 6. It is seen that regardless of the materials, these parameters indicate approximately a linear relationship on the logarithmic scale.

The various methods to determine $(Du_f/Dt)^*$ are considered. Between time $t_1$and t_2, we assume that the slope changes linear. The predicted value can be expressed as,

$$(Du_f/Dt)^* = (Du_{f1}/Dt) + \frac{(Du_{f2}/Dt) - (Du_{f1}/Dt)}{\Delta t} \times \Delta t_1 \quad (9)$$

where $\Delta t = t_2 - t_1$. When $\Delta t_1 = 0.5\Delta t$, it is a simple average value of the two known slope. We call this case is 'simple averaging' here. In addition, two types of prediction methods for $(Du_f/Dt)^*$ were examined. These employ interpolation function, and these are,

$$(Du_f/Dt)^* = Z_1 \exp(Z_2 \overline{u}_f) \quad (10)$$

$$(Du_f/Dt)^* = k_1 \exp(k_2 t^*) \quad (11)$$

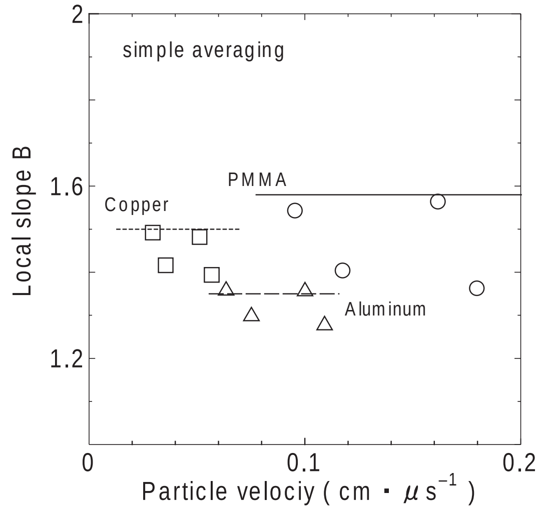


Fig. 7 The local slope of the Hugoniot obtained by proposed method (simple averaging) The straight lines are an actual slope B.

Table 2 The predicted local slope B and the difference between predicted and actual B
B was predicted by using the information of 10 and 15mm from the explosive, and Eq.11 was applied for our prediction methods. A parenthesis is the thickness of C4.

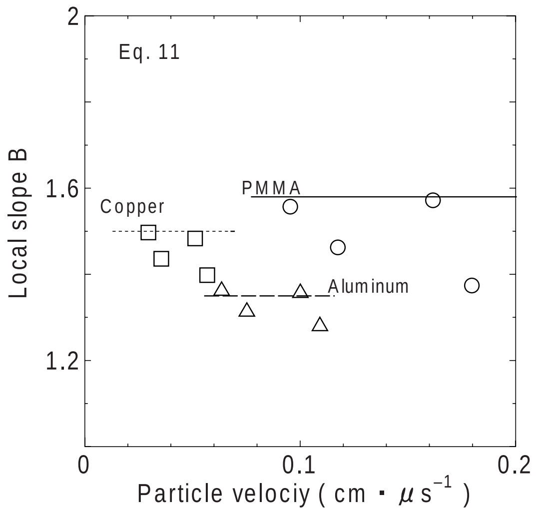
Materials	PA(3)	PA(15)	AL(3)	AL(15)	CU(3)	CU(15)
This work	1.5559	1.5708	1.3617	1.3568	1.4959	1.4821
Difference (%)	1.43	0.49	0.87	0.50	0.07	0.99

The constants Z_1, Z_2, k_1, and k_2 were determined by the slope of the particle velocities at both side. Because the shock wave attenuates with the propagation, the relation of $\Delta t_1 < \Delta t_2$ can be considered. The time in Eq. (11) was estimated by $\Delta t_1/\Delta t_2 = u_{f2}/u_{f1}$.

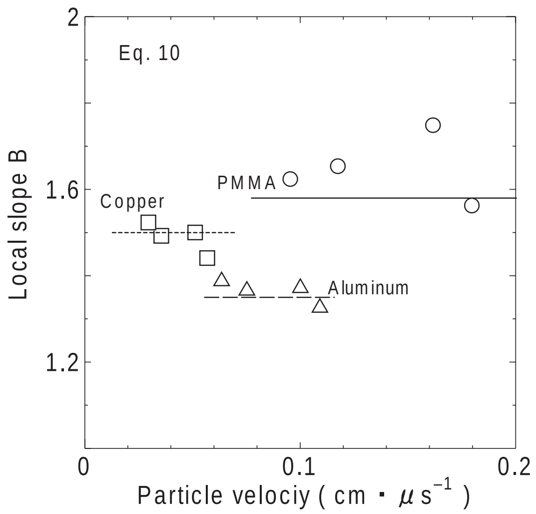
The local slope of the Hugoniot obtained by the above mentioned $(Du_f/Dt)^*$ are shown in Figs 7 and 8. The straight lines correspond to actual B for PMMA, Aluminum, and Copper. The points away from the lines are a local slope of Hugoniot predicted by using the information of 5 and 10 mm from the explosive. It is considered that since the change of the slope of the particle velocity is large in this region, the accurate prediction of the slope of the particle velocity between two points is difficult in the current method. Except those points, the proposed method gives the good results for prediction of the local slope of the Hugoniot as shown in Table 2. This study has demonstrated that the differences between the predicted and the actual values of the slope B is less than 1.0 % when the measured point is put on between 10 and 15 mm from the explosive.

5. Conclusion

A new method for estimating the Hugoniot of the materials was proposed and was based on the exact formulation of the shock wave decay process. The feasibility of the proposed method was investigated using the results of the numerical simulations which were assumed as gauge records. Even in the case of an attenuating shock wave, the



(a) Using $(Du_f/Dt)^*$ as exponential function of t



(b) Using $(Du_f/Dt)^*$ as exponential function of $\overline{u}_f$

Fig. 8 The local slope of the Hugoniot obtained by proposed method The straight lines are an actual slope B.

Hugoniot can be determined with high accuracy using the averaging shock velocity and the particle velocity. Using those average values our proposed method can predict the local slope of the Hugoniot. This study has demonstrated that the differences between the predicted and the actual values of the slope B is less than 1.0 % when the measured point is put on between 10 and 15 mm from the explosive. However, near the explosive, the improved prediction method for determining the local slope of profiles of particle velocity must be constructed as future work.

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新しい考え方によるHugoniotの評価法

久保田士郎*†，佐分利禎*，緒方雄二*，永山邦仁**

衝撃波伝播を記述する非定流れの式に基づく材料のHugoniotを見積もるための新しい方法を提案した。一般に，Hugoniot測定は完全に矩形状の衝撃波を使用するが，減衰する衝撃波の場合であっても，本方法を適用できる。本手法は二つの粒子速度履歴を用いて，Hugoniotの関係，およびHugoniotの局所的な勾配を決定できる。

*独立行政法人　産業技術総合研究所　つくば西　安全科学研究部門　爆発安全研究コア
〒305−8569　つくば市小野川16−1
†Corresponding addres : kubota.46@aist.go.jp

**九州大学
〒819−0359　福岡市西区元岡774番地