A simple method of deducing JWL parameters from cylinder expansion test

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A simple method has been developed for deducing the JWL equation of state (EOS) of high explosive detonation products from the cylinder expansion test. This method, which is significantly simpler than that of a large computer hydrocode, still retains the same degree of accuracy.

The radial expansion history of metal tube, which is recorded by the streak camera, is expressed as an appropriate fitting function. The p−V relation of detonation products may be obtained from the differentiation of fitting function and equations of conservation law. Metal strength is also taken into consideration to reduce the deviation at the lower pressure region. JWL parameters are acquired through the p−V relations by another nonlinear curve fitting procedure.

The computed results for Comp−B, TNT, HMX, PBX−9404 and Nitromethane, which are in good agreement with Lawrance Livemore National Laboratory (LLNL) data.

1. Introduction

Research and design of explosive devices would usually use hydrocodes. The accuracy of simulation depends highly on the equation of state of the detonation products used for the high explosive. A certain number of EOS exist which express the detonation product isentropes, such as polytropic gas law, LJD (Lennard−Jones−Devonshire), BKW (Becker−Kistiakowsky−Wilson), and JWL (Jones−Wilkins−Lee). JWL EOS is generally more common than other EOSs, especially in the hydrodynamic calculation. The equation contains sixparameters, describing the relationship among the volume, energy and pressure of detonation products. These parameters may be determined by the cylinder expansion test.[1][2].

LLNL obtain these six JWL parameters by a trial and error procedure. The expansion record, simulated by a hydrocode, was repeatedly calculated in this try and error procedure until the calculated history of radial displacement matched with the experimental data. This procedure was iterative and time−consuming, and some simpler methods were then presented for saving time. An analytical model was used by Polk [3], which did not need a hydrodynamic code. The p−V relationship was then calculated directly from the expansion history of metal tube. A certain equations proposed in his model, are liable to be invalid, for example oscillation may occur at the lower pressure region. A two−dimensional characteristics code coupled with nonlinear curve fitting technique was proposed at the same time by Bailey[4]. The calculation was made easier than that of LLNL. Later, a new and simple model combined with curve fitting procedure developed by Hornberg[5] possesses the characteristic of good accuracy. Meanwhile, Ijsselstein et al. [6]. proposed a trial and error procedure with

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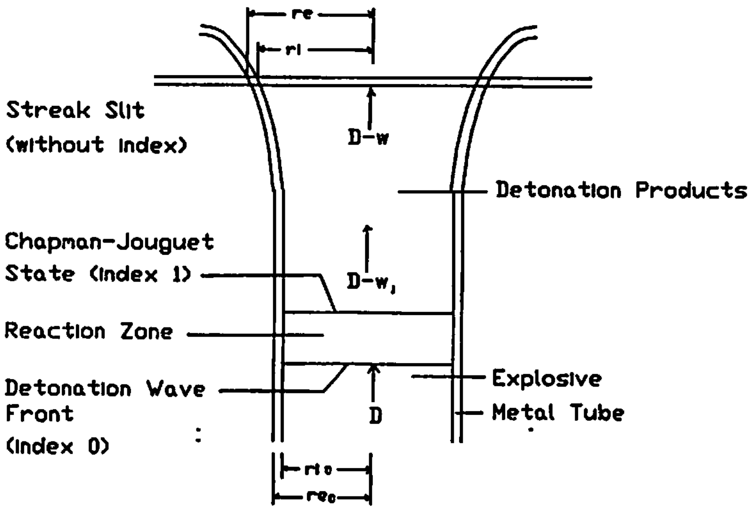

Fig. 1 Flopw conditions of the detonation products in expanded metal cylinders

simple model instead of hydrodynamic code. It was however, still an iterative algorithm, as the above paragraph has illustrated. Polk's algorithm has been the simplest, but also relatively inaccurate. In addition, the simple model proposed by previous literature has been either too simple and neglects accuracy or the deducing equations are inconsistent with the practical phenomena of shock physics.

The principal aim of this paper is therefore to set up a simple and accurate analytical method, to determine the parameters of JWL. The accuracy of this method can keep up with the large hydrocodes. A metal strength[7] is, additionally, introduced here to get rid of the deviation at the low−pressure region.

2. Theoretical analysis

The following assumptions are made in our derivation of the relationship between pressure and volume of detonation products.

(A) Detonation wave is one dimensional and is in a steady state.

(B) The explosive is transformed instantaneously to detonation products.

(C) The copper tube is incompressible.

(D) The reverberation of shock wave in metal tube is neglected.

The image of expansion tube recorded by the streak camera is the outer radius of copper tube, r_e. Gas pressure, in practice, directly exerts itself on the inner surface with the radius r_i, and particle motion may than be represented by a mass point. Both velocity and acceleration of the particle are therefore set on a mass point which is the central surface with the radius r_m. The relation among the r_e, r_m and r_i is given, according to assumption (C), as

$$(r_e^2-r_m^2)=A_o/(2\pi)=(r_m^2-r_i^2) \qquad (1)$$

where A_o is the cross section of copper tube, which is equal to $\pi\ (r^2_{eo}-r^2_{io})$; r_{eo} and r_{io}, meanwhile, denote the initial outer and inner radii. The geometric relation among the various radii is shown in Fig. 1.

2.1 Radial displacement function of the copper tube

A nonlinear curve fitting procedure[8] is adopted here to attain an appropriate function representing expermential data. The fitting function must satisfy the following constraints since it describes the radial displacement of metal tube impelled by product gases.

(A) Fitting function must show zero intercept, because initial displacement is zero.

(B) Initial slope is zero, because velocity is zero at the beginning.

(C) Slope of final expansion stage is finite, which corresponds to the terminal velocity of the tube wall.

(D) Initial expansion pressure is the CJ pressure.

The exponential drop in the pressure behind the detonation front is assumed at first. The following expression is then obtained here

$$\frac{d^2r_m}{dt^2}=\sum_{j=1,n} a_j b_j^2 \exp(-b_j t) \tag{2}$$

In order to avoid the probability of zero denominator after integrating Eq. (2), b is written as square.

Eq. (2) can be respectively taken to the first and second integration, under the constraints of (A), (B) and (C), then

$$\frac{dr_m}{dt}=\sum_{j=1,n} a_j b_j(1-\exp(-b_j t)) \tag{3}$$

and

$$r_m-r_{mo}=\sum_{j=1,n} a_j\,(b_j t-(1-\exp(-b_j t))) \tag{4}$$

Both Bailey[4] and Lee[9] indicate that cylinder motion is unstable at the early stages of expansion. so the instant of initial expansion of copper tube is difficult to recognize. Whether this time is either too early or too late will give rise to a shifiting of leading or delay in time coordinate. In addition, the experimental data of LLNL, shown in Table 1. do not satisfy constraint (A) which requires that $r_m - r_{mo} = 0$ when $t = 0$. We

Table 1 Measurement values of expansion history in LLNL

r_e-r_{eo}	t (μ sec)				
	Comp-B	HMX	TNT	PBX-9404	Nitromethane
0.5	0.7	0.52	0.92	0.57	1.2
1	1.27	1.0	1.545	1.09	2.02
1.5	1.71	1.39	2.13	1.48	2.72
2	2.11	1.79	2.64	1.87	3.35
2.5	2.59	2.16	3.15	2.23	3.95
3	2.99	2.49	3.615	2.59	4.5
3.5				2.93	
4	3.78	3.13	4.53	3.26	5.56
5	4.52	3.75	5.41	3.91	6.55
6	5.22	4.35	6.25	4.56	7.52
7	5.91	4.94	7.06	5.18	8.46
8	6.59	5.52	7.85	5.78	9.37
9	7.26	6.09	8.63	6.38	10.26
10	7.92	6.66	9.41	6.97	11.14
11	8.57	7.22	10.18	7.56	12.0
12	9.22	7.77	10.94	8.14	12.86
13	9.86	8.32	11.69	8.71	13.72
14	10.5	8.87	12.44	9.28	14.57
15	11.13	9.41	13.17	9.85	15.42
16	11.75	9.95	13.9	10.41	16.26
17	12.37	10.49	14.62	10.97	17.09
18	13.0	11.03	15.34	11.53	17.91
19	13.6	11.57	16.05	12.08	18.73
20	14.22	12.1	16.76	12.64	19.54
21	14.83		17.47		
22	15.43	13.17	18.18	13.57	21.17
23	16.04				
24	16.64	14.23	19.59	14.85	22.78
25	17.24	14.75	20.25		23.58
26	17.84			15.94	
28	19.04			17.03	
30	20.23			18.11	
32	21.42				
34	22.6				

therefore introduce a fitting constant, t_o, to represent the instant of initial expansion. After $t' - t_o$ is substituted for t in Eq. (4), we get

$$r_m - r_{mo} = \sum_{j=1,n} a_j (b_j (t'-t_o) - (1-\exp(-b_j(t'-t_o)))) \tag{5}$$

where t' represent the experimental data t in Table 1.

In order to have enough parameters for curve-fitting, the n in Eq. (5) is taken as 2. The fitting parameters of some explosives are shown in Table 2.

2.2 Particle velocity and acceleration in metal tube

The $r_m(t) - r_{mo}$ with respect to time, t, is the reading displacement of copper tube with the streak camera and not a trace of the motion of an individual particle. This reading may be viewed as the Eulerian coordinate system but the motion of actual wall particle is described more properly by Lagrangian than by Eulerian system. Transferring from the Eulerian system to the Lagrangian system is then necessary.

First, let z and r represent the axial and radial coordinate in Eulerian system, with $z=0$ at the observation position and $r=0$ along the central region of cylinder. The cylinder particle is flowing with the constant velocity, D, in the longitudinal section of the cylinder, when the coordinate system is fixed on the detonation front.

The expanding path of individual cylinder particle is shown in Fig. 2(a). The corresponding positions along the cylinder wall can be seen to be at the P' and Q' while P and Q are recorded by the camera at the time t_1 and t_2. After t_1 and t_2 in time occurs, the detonation front occupies new positions where $z_1 = Dt_1$ and $z_2 = Dt_2$ respectively. Both P' and Q' have actually already ex-panded outwardly before both P and Q have been recorded by a camera ; the time, namely τ, of the beginning of particle motion is then larger than t, which is measured by the camera.

Because of the assumptions of incompressiblity of cylinder wall and negligible pressure in axial direction, the detonation wave has no influence on the length of copper tube. The are length of s (t_1) shown in the Fig. 2 (b) must also be equal to the incremental distance swept by the detonation wave during the interval τ. The radial and axial coordinates of point P, g (τ) and f (τ), may then be represented by

$$g(\tau) = r_m(t) - r_{mo} \tag{6}$$

$$f(\tau) = D(\tau - t) \tag{7}$$

From Fig. 2 (a) it is seen that

$$\frac{d\tau}{dt} = [1 + ((1/D)dr_m/dt)^2]^{1/2} = 1/\cos\theta \tag{8}$$

and

$$dr_m/dt = D\tan\theta \tag{9}$$

The velocity and acceleration measured at the central surface are

$$v = ((dg/d\tau)^2 + (df/d\tau)^2)^{1/2} \tag{10}$$

and

$$a = ((d^2g/d\tau^2)^2 + (d^2f/d\tau^2)^2)^{1/2} \tag{11}$$

Furthermore, using L'Hospital rule gives :

$$dg/d\tau = (dr_m/dt)\cos\theta \tag{12}$$

$$df/d\tau = D(1-\cos\theta) \tag{13}$$

$$d^2g/d\tau^2 = (d^2r_m/dt^2)\cos^4\theta \tag{14}$$

$$d^2f/d\tau^2 = \sin\theta\cos^3\theta(d^2r_m/dt^2) \tag{15}$$

Velocity, v, and acceleration, a, are then obtained by substituing Eqs. (12) and (13) into Eq. (10), and Eqs. (14) and (15) into Eq. (11).

2.3 Copper tube expantion

The wall material flows at the constant velocity D in the longitudinal section of the cylinder, according to Taylor[10]. This is viewed from the position of the system fixed in the detonation front. The movement

Table 2 Curve-fitting prameters for the cylinder expansion

Explosive	A_1	A_2	B_1	B_2	t_0
Comp B	3.67494	1.11089	0.17583	0.96578	0.5028
HMX	2.25908	0.25491	0.61694	1.71148	0.2924
TNT	2.03321	0.28403	0.60449	0.56745	0.07122
PBX 9404	2.80653	0.2975	0.47832	1.6382	0.40729
Nitromethane	3.41908	0.52548	0.25048	0.71965	0.4264

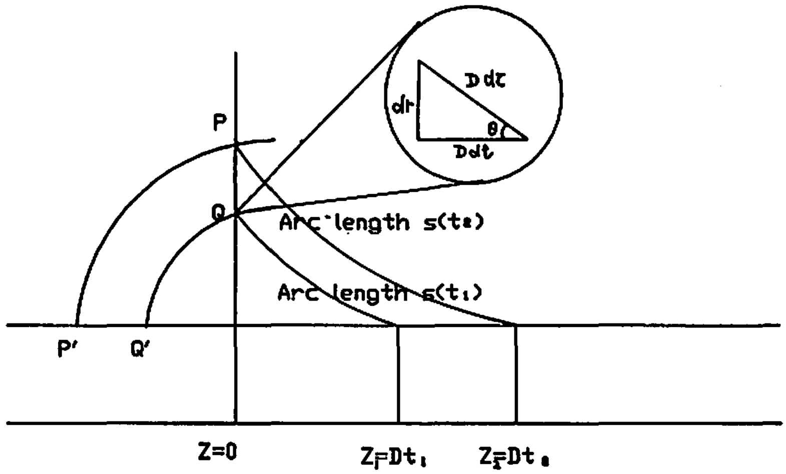

Fig. 2(a) Particle trajectory in time scale t

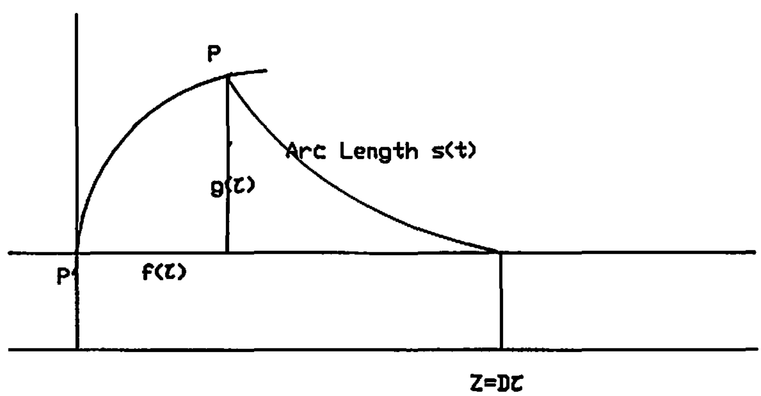

Fig. 2(b) Particle trajectory in time scale τ

in the expansion area behind the detonation front then, as a result, takes place in a convexly bent curve. This consequently results in a centrifugal acceleration, a, being directed towards the center of the cylinder. The production of the centrifugal force Ma then results due to this, where M is the cylinder mass per unit of length. This latter has to be compensated by pressure p of the detonation products or the contracting force $2\pi r_i p$ on account of the stationary :

$$p = \text{Force/Area} = (M\,a)/(2\pi\, r_i) \tag{16}$$

This expression can be carried over by the work–energy relation

$$M\,a\,dr_i = (1/2)M\,dv^2 \tag{17}$$

or

$$a = \frac{1}{2}\,\frac{dv^2}{dr_i} \tag{18}$$

Eq. (18) may therefore be integrated in the form

$$v^2 = \frac{2\pi}{M}\int_{r_{io}}^{r_i} p\,dr_i^2 \tag{19}$$

Referring to Taylor's derivation, in our notation, Eq. (19) can be integrated by parts when the Bernoulli's equation and continuity equation are taken into account. Then

$$v^2 = \frac{2D}{\mu}\left(\frac{\rho}{\rho(D-w)} - w\right) \tag{20}$$

where μ is weight of cylinder per weight of explosive. The equation of continuity, by referring to Fig. 1, is written as

$$\rho_o D r_{io}^2 = \rho(D-w)r_i^2 \tag{21}$$

or

$$V = \frac{\rho_o}{\rho} = \frac{D-w}{D}\left(\frac{r_{io}}{r_i}\right)^2 \tag{22}$$

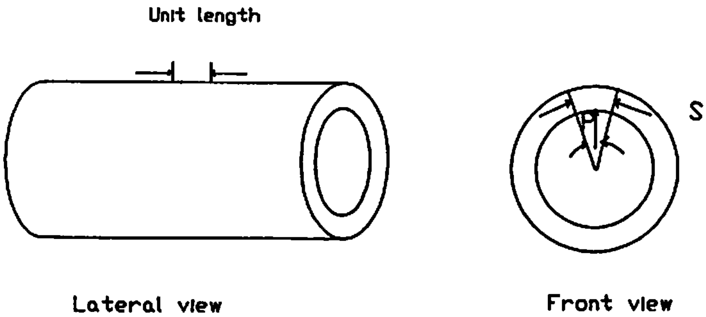

Fig. 3 Thin ping configuration

Table 3 Detonation properties for explosive interested

Explosive Composition (%)	ρ_0 (g/cm³)	D (m/s)	P_I (GPa)	$E_0=\rho_0(e_0+q)$ (GPa)
Comp B 64 RDX/ 36 TNT	1.717	7980	29.5	8.5
HMX	1.891	9110	42.0	10.5
TNT	1.63	6930	21.0	7.0
PBX 9404 94 HMX/3 NC/3 CEF	1.84	8800	37.0	10.2
Nitromethane	1.128	6287	12.5	5.1

Substituting Eq. (21) into Eq. (20) becomes

$$w=\frac{1}{\rho_o D}(p(r_i/r_{io})^2-(\mu/2)\rho_o v^2) \quad (23)$$

2.4 Metal strength

Metal strength, which is relatively small in comparison with an extremely high detonation pressure at the initial stage, has no obvious effect. The metal strength may, however, influence the pressure calculation at the final expansion stage.

To take into account of metal strength, consider the case of a thin circular ring subject to the action of uniformly distributed radial loading as shown in Fig. 3 If the cross sectional area of the ring is constant along the circumference and the thickness Δr is small compared with the radius r_m, such loading will produce uniform stress and axial stress can be neglected, then the equation of motion of the ring may be written as the following

$$\rho_m r_m d\alpha \Delta r a = p r_i d\alpha - S d\alpha \quad (24)$$

or

$$\rho_m r_m a = p r_i/\Delta r - \sigma \quad (25)$$

After rearrangement,

$$\begin{aligned} p &= \rho_m (r_m/r_i)\Delta r a + \sigma(\Delta r/r_i) \\ &= Ma/(2\pi r_i) + \sigma(r_e^2-r_i^2)/(2r_m r_i) \end{aligned} \quad (26)$$

and

$$\sigma=E_t=E(r_m-r_{mo})/r_{mo}$$
$$\text{If } \sigma>\sigma_\gamma, \text{ then let } \sigma=\sigma_\gamma \quad (27)$$

where ρ_m, σ, σ_γ, E and S indicate the density of metal, hoop stress, yield stress, Young's modulus and hoop strength respectively. In this paper E = 103.35GPA, σ_γ = 0.31GPA.

2.5 JWL parameters calculation

Given the experimental date $r_e(t)$ with respect to t, the p−V relationship of detonation product gases may be obtained by the nonlinear curve fitting, conservation equations, and metal strength. However, in the warhead design, the p−V relationship must transform to an analytical form to be conducive to calculating in hydrodynamic code. JWL is discussed here, which gives

$$p(\Delta)=Ae^{-R_1V}+Be^{-R_2V}+C/V^{\omega+1} \quad (28)$$

where A, B, C, R_1, R_2 and ω are the parameters also obtained by the nonlinear curve fitting with three specific constraints[5], $P=P_{cj}$, $E=E_{cj}$, $V=V_{cj}$, in the Chapman−Jouget condition.

Table 3 shows a certain detonation properties for the explosives interested.

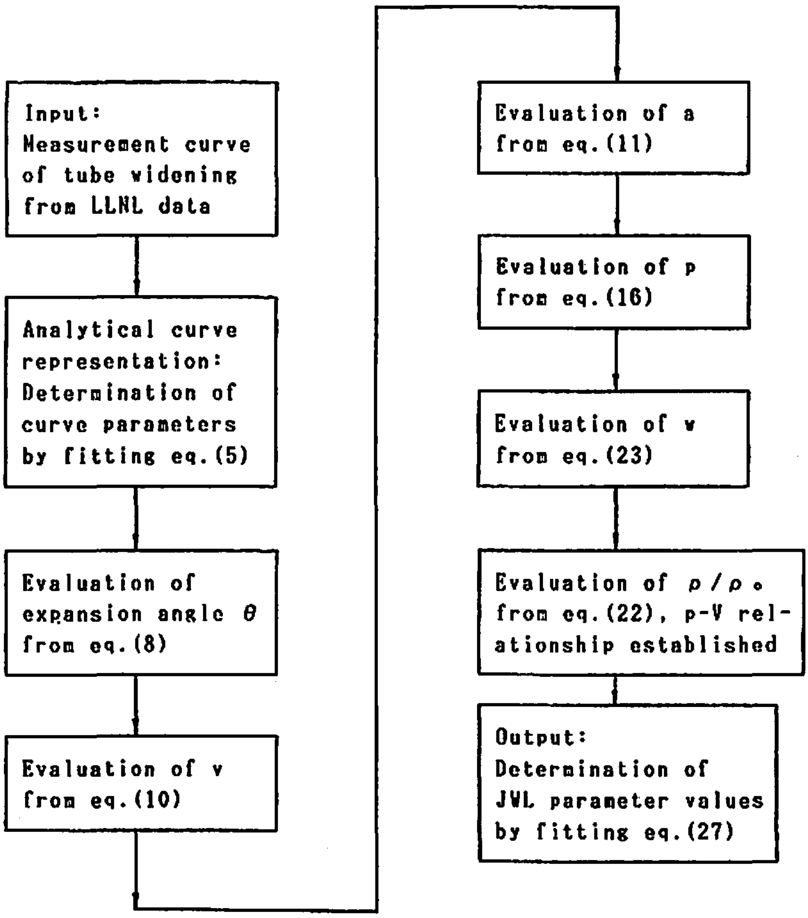

Fig. 4 Flow chart of our calculation procedure

The whole process of the calculation is summarized in the flow chart shown in Fig. 4.

3. Results and Discussion

The radial expansion of the Comp−B copper tube recorded by LLNL streak camera is shown in fig. 5. The curve is fitted from these radial expansion data in Table 1 and represented by Eq. (5). The fitting parameters in Eq. (5) are shown in Table 2. It can be seen that, the curve indeed agree with the constraints $r_m - r_{mo} = 0$ at $t' - t = 0$ or $t = 0$.

Fig. 6 shows the p−V data obtained by LLNL and our method. The data with symbol Δ represents JWL EOS proposed by LLNL, the other data with symbol □ and ○ represent the result of our method not including metal strength, and our method taking metal strength into cənsideration, respectively. It can be seen that the effect of metal strength obviously increases the pressure considerably in the low pressure

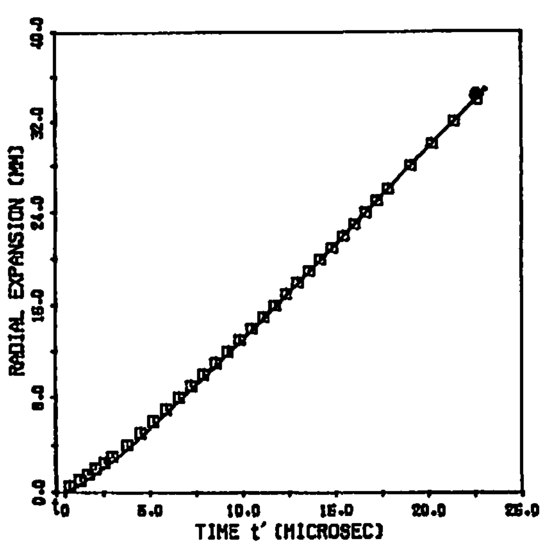

Fig. 5 Comparison of experimental data of expansion tube in LLNL and fitting curve deduced by this paper

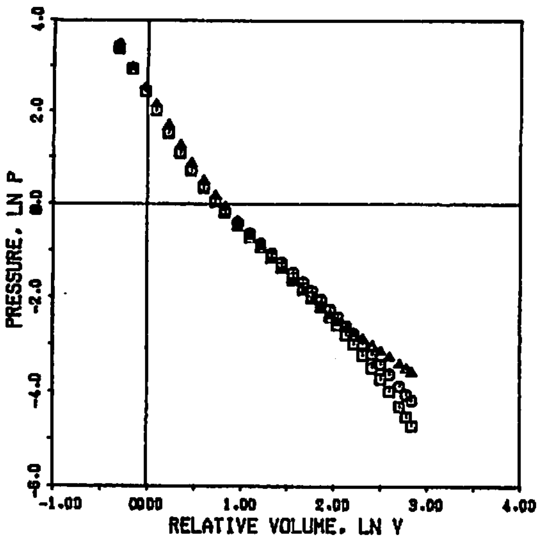

Fig. 6 Comparison of p-V data for comp-B calculated by our method including metal strength (○) and not including the metal strength (□) and LLNL (△)

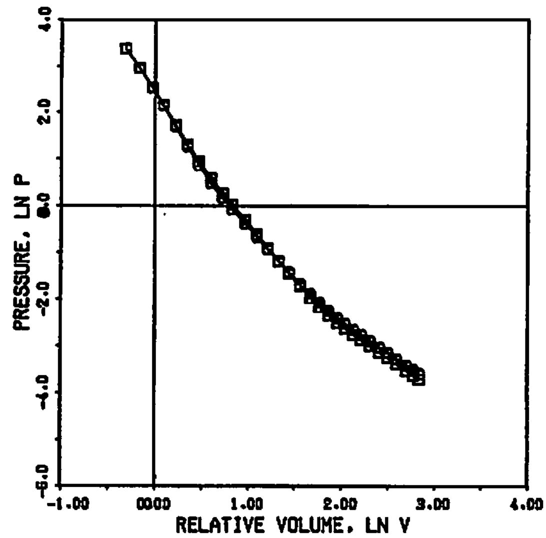

Fig. 7 Comparison of p-V curve for comp-B obtained with our method (○) and LLNL (□)

Table 4 JWL parameters composition with those found in literature

Explosive		A (mbar)	B (mbar)	C (mbar)	R_1	R_2	ω
Cpmp-B	(1)	5.24229	0.076783	0.010818	4.2	1.1	0.34
	(2)	5.03808	0.05304	0.01183	4.1	1.05	0.33
	(3)	4.96376	0.03944	0.01288	4.06244	0.94846	0.35
	(4)	5.2323	0.06793	0.009667	4.169	1.042	0.33
HMX	(1)	7.78280	0.070714	0.00643	4.2	1.0	0.30
	(2)	7.736	0.10467	0.0089	4.169	1.036	0.33
	(3)	7.7831	0.16808	0.01075	4.27291	1.33644	0.30
TNT	(1)	3.71213	0.032306	0.010453	4.15	0.95	0.3
	(2)	3.74911	0.07542	0.01178	4.24123	1.56431	0.3
	(3)	3.62033	0.02492	0.00887	4.07257	0.88784	0.25
PBX 9404	(1)	8.524	0.18020	0.01207	4.60	1.30	0.38
	(2)	8.8704	0.21591	0.01124	4.69866	1.35283	0.38
	(3)	7.46850	0.06909	0.01447	4.26914	1.05592	0.35
Nitro-methane	(1)	2.0925	0.056895	0.007704	4.40	1.20	0.30
	(2)	2.09197	0.04608	0.00795	4.33414	1.13408	0.3
	(3)	2.03532	0.003605	0.00984	4.28602	1.08386	0.35
	(4)	2.1348	0.071334	0.008641	4.5188	1.3458	0.4

(1) LLNL
(2) This paper
(3) Hornberg
(4) Ijsseistein

region.

Fig. 7 are the p−V curves fitted by our p−V data including metal stregth effect and proposed by LLNL, and the closeness between two curves demonstrate that our proposed method including metal strength can be accurate and timesaving and it gives the equation of state for detonation products. Listed in Table 4 are the JWL parameters for various explosives computed by the aforementioned methods. It can be seen that our method has indeed improved the Hornberg's calculation and has about the same accuracy as Ijesslestin's.

4. Conclusion

Laborious iteration often used in hydrodynamic code calculation has been ommited here so that our method achieves approximately the same accuracy as the result given by the huge hydrodynamic code of LLNL. The oscillation found in Polk's computation has been eliminated.

Based on detonation physics adn non−linear curve fitting, our method first calculates the function which represents the copper tube radial displacement $r_m - r_{mo}$ with respect to time t, then the p−V relationship, and finally the parameters of JWL equation. Taking into consideration the metal strength, this metod improves the accuracy in the low−pressure region.

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シリンダー膨張試験を用いたJWL状態方程式パラメータの簡便推算法

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高性能爆薬類の爆轟に関するJWL状態方程式のパラメータを求める方法として，シリンダー膨張試験を用いた簡便法を開発した。この方法は，大規模なコンピュータコードを用いるよりもはるかに単純であるが，得られたパラメータはそれらと同程度の精度を有している。ここでは，この方法を概説するとともに，コンポジションB, TNT, HMX, PBX-9404，ニトロメタンのJWL状態方程式パラメータについて，本法によって得られた値と，従来法によるものとの比較を示した。

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