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THEORY OF RELATIONS BETWEEN THE DETONATION VELOCITY OF SOLID EXPLOSIVES AND THE THICKNESS OF CASES OR THE DIAMETER OF THE CHARGES

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Abstract

A theoretical formula has been presented, which describes relations among the maximum detonation velocity D_m, any stable detonation velocity D, reaction time t_0 in (detonation) reaction zone, the thickness of wall Y, the velocity of the shock waves in the wall material v_w and the maximum reaction time t_m, based on the assumption that the duration of the reaction time is elongated by the existance of the solid wall, where the wall begins to move only after a shock wave, whose velocity is v_w, has travelled from the inner surface to the outer one. The assumption is consistently applied to both the so-called thin and thick wall. Also a theoretical formula has been presented, which describes relations between D_m, D, radius of a cartridge R, critical radius R_c, below which no detonation occurs, t_m, the critical reaction time t_c, the velocity of sound in detonation products v_c, based on the assumption that the part of a charge around the critical diameter acts as a case. A general equation, in which both the thickness of a case and the diameter of charges vary, has deen constructed superposing the two cases described above.

The validity of the fundamental formulas, which contain no arbitrary constant in contrast to the previous formulas described by

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various authors, have been verified by a series of experimental data due to Copp, Ubbelohde, Cook et al.

t_m, t_c, the maximum reaction zone length X_m, the critical reaction zone length X_c have been calculated by use of these formulas, which may be called "the variable reaction zone length theory". X_m, T_m are found to be much longer than have been supposed to be by many previous theories.

The general equation derived by the present theory is:

$$\left(\frac{D}{D_m}\right)^2=\left\{\left(t_c-\frac{R_c}{v_e}\right)\frac{1}{t_m}+\frac{Y}{v_w}\frac{1}{t_m}\right\}+\left(\frac{1}{v_e t_m}\right)R,\ t_0=t_c+\frac{R-R_c}{v_e}$$

§. 1. Introduction

(1) The theories proposed by H. Jones[1)2)3)] and H. Eyring[4)] assume the constancy of the length of the reaction zone X while in the present theory which may be called "the variable reaction-zone-length theory," the length of the reaction zone is assumed to vary from the maximum reaction zone length X_m, which corresponds to the maximum detonation velocity D_m, to the minimum or critical reaction zone length X_c. The reaction time is also assumed, in the present theory, to vary from the maximum t_m to the minimum or critical t_c.

(2) The theories of Jones and Eyring assume that the thickness of casings Y or the radius of charges R change the mass equation and the momentum equation among the fundamental equations of the theory of detonation while in the present theory only the energy equation is influenced by Y and R.

(3) In the theories of Jones & Eyring no straight-forward equations between D and Y or R are deduced in analytical forms and a number of practical formulas of limited application have been suggested based on numerical approximation, that is, the results were semi-empirical equations containing some arbitrary constants whose physico-chemical meanings are not necessarily clear while in the present theory the equations for D, Y and R contain no arbitrary constant and the variables contained in the equations have definite physico-chemical meanings and can be determined numerically by a series of experiments on D and Y or R.

(4) In both classical theories the functions of the so-called thin wall and the thick one are different. In the former the inertia of the wall plays a main part and in the latter case the shock wave within the wall plays a main part while in the present theory the effects of confinement in both cases are due to the action of the shock wave, however thin the wall may be. And the effect of R on D is also interpreted as a confining action of a sheath of an explosive charge around "the critical radius of a cartridge."

The defects (1)∼(4) of the classical theories have correlations. Once we assume (1) a constant value of X (and t), which means a fixed position of Chapman-Jouguet plane, then (2) the Hugoniot energy equation can not be modified drastically and (3) the calculations become complicated in

1) H. Jones: Proc. Roy. Soc., A. **189** pp. 415–426 (1947).

2) John L. Copp and A. R. Ubbelohde: Trans. Faraday Soc. XLIV. 646–669 (1948).

3) J. Taylor: Detonation in Condensed Explosives: Oxford, The Clarendon Press, p. 150 (1952).

4) Henry Eyring, Richard E. Powell, George H. Duffey, Ransom B. Parlin: Chemical Reviews, **45**, 69–179 (1948).

dealing with the two equations, the mass and momentum equations, instead of one energy equation and (4) to explain the effects of the radius, the thickness of cases different mechanisms and different equations must de introduced for thin and thick walls because X (and t) is assumed constant. In contrast to this, the present theory assumes that X (and t) varies thus leading to one mechanism or a general equation which covers all effects of the radius, thin and thick walls.

(5) R. Schall[5] discussed the stability of lower detonation velocity on the hypothesis that X is proportional to $1/D$ while in the present paper X is assumed to be increasing with D^2.

(6) M. A. Cook[6] introduced the idea of the detonation head or the geometrical model and assumed that

$$t=4cd'/3D$$

where c is a proportionality constant between zero and 1.0 in an assumption $h=cd'$=length of the detonation head, and d' is empirically found to be: $d'=2R-0.6$ cm. In the present theory no arbitrary constant has been used in deriving the theoretical formulas.

(7) N. Manson[7] discussed the effect of R on D on the basis of the critical radius below which no detonation occurs although his theory did not give any description on the structure of the reaction zone while in the present theory the critical radius plays a definite part in the structure of the reaction zone.

5) Rudi Schall: Zeitsch. für angewandte Physik, **6**, 470–475 (1945).

6)-1. Melvin A. Cook, G. Smoot Horsley, W. S. Partridge, W. O. Ursenbach: J. Chem. Physics. **24**, 60–67 (1956).

6)-2. M. A. Cook, Earle B. Mayfield, W. S. Partridge: J. Phy. Chem. **59**, 675–680 (1955).

6)-3. M. A. Cook, Ferron A. Olson: A. I. Ch. E. Journal. **1**, 391–400 (1955).

7) N. Manson: Zeitschrift für Elektrochemie: **61**, 586–592 (1957).

§. 2. Theory on the effect of an enclosure with the thickness Y

§. 2-1. Assumption of reaction zones with variable lengths and the stability of detonation

For a stable detonation with the detonation velocity D there is an approximation equation[8] between the detonation velocity D_m and the heat of detonation Q_m

$$D_m \doteqdot \frac{v_0}{v_0-\alpha}\sqrt{2Q_m(k_1^2-1)} \quad (2\text{-}1)$$

If we put:

Q_e=effective detonation heat available on detonation passage in the case of non-ideal detonation,

then the observed detonation velocity D is determined not by Q_m but by Q_e, that is, Q_m in the Hugoniot energy equation should be replaced by Q_e,[9] while Q_e depends on the degree of confinement which in turn influences the observed D.

or
$$\frac{D}{D_m}=\sqrt{\frac{Q_e}{Q_m}} \quad (2\text{-}2)$$

$$\frac{Q_e}{Q_m}=N=\text{fraction of heat utilized on detonation passage} \quad (2\text{-}3)$$

In other words, to keep a detonation wave at a constant velocity D the consumption of the effective energy Q_e is required, therefore,

8) (3) p. 89 Table 22.

9) Kumao Hino: Detonation velocity of explosives. Journal of the Industrial Explosives Society, Japan. (In Japanese) **8**, 66–74 (1948), **9**, 9–21, 47–60 (1948).

(2-2) may be described as "the energy consumption relation."

Although there is no limitation to the magnitude of reaction zone length X (and reaction time t) from the standpoint of the thermodynamical-hydrodynamical theory of detonation, reaction zone should have some definite length X for a given explosive cartridge under a given condition (case, radius, loading density). Below this certain value of X the effective heat evolution may not be enough to support the stable detonation while above this value of X the expansion of the decomposed explosive makes the elongation of X impossible. X can vary over a wide range when cases or the radius change. Under a given condition:

$$X=\text{const.}=X_1 \qquad (2\text{-}4)$$

Reaction time t is described as follows;

$$t=X/D \qquad (2\text{-}5)$$

Then for a fixed value of X_1

$$t_1=X_1/D \qquad (2\text{-}6)$$

Let us assume, to a first approximation, that the fraction of the effective detonation heat available on detonation passage is' proportional to the reaction time t.

$$N=\frac{Q_e}{Q_m}=\frac{t}{t_m} \qquad (2\text{-}7)$$

where: t_m=maximum reaction time.

As the total energy is utilized at $X=X_m$, X_m becomes the same with "the constant reaction-zone-length" assumed in the classical treatment of Jones and Eyring.

Combining (2-5) with (2-7):

$$N=\frac{X/D}{X_m/Dm}=\left(\frac{X}{X_m}\right)\left(\frac{D_m}{D}\right) \qquad (2\text{-}8)$$

For a given explosive, X_m and D_m have fixed values independent of the degrees of the confinement and the radius of charges. Therefore

$$\left(\frac{D_m}{X_m}\right)=\text{const.} \qquad (2\text{-}9)$$

Or for a given explosive under a given condition:

$$N=\left(\frac{X_1 D_m}{X_m}\right)\frac{1}{D}=\frac{\text{const.}}{D} \qquad (2\text{-}10)$$

(2-10) gives "the energy supply relation." When "the energy consumption relation" (2-2) and "the energy supply relation" (2-10) are balanced at point P_1 in Fig. 1. the stability of detonation is obtained because if we increase the detonation velocity D, by a small amount, then the energy consumption exceeds the energy supply and D begins to decrease and vice versa.

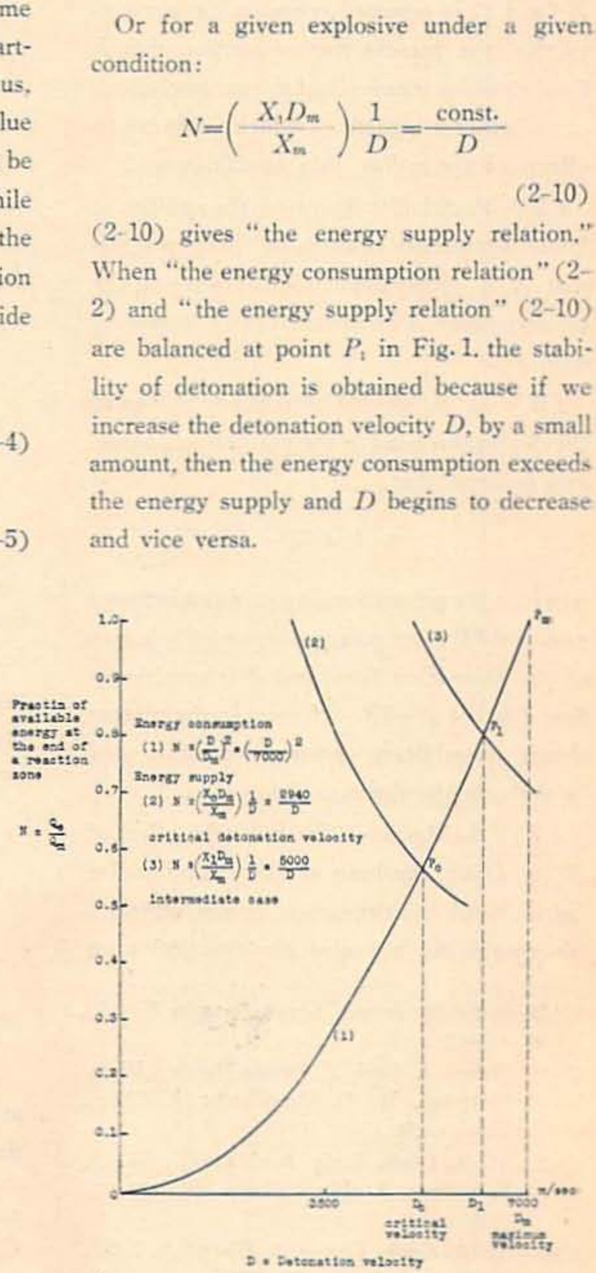

Fig. 1. Stability condition of detonation

In Fig. 1. the curve (1) shows "the energy consumption relation," while curves (2) and (3) show "the energy supply relations" for different reaction zone lengths. The point P_m indicates the case of the ideal detonation velocity where maximum detonation velocity D_m is obtained with maximum reaction zone length X_m (and the maximum reaction-time t_m). The curve (3) shows an intermediate case, where by a poor confinement or by a smaller radius of a charge, an intermediate detonation velocity D_1 is obtained with a shorter reaction zone length X_1 (and a shorter reaction-time t_1). The curve (2) represents the case where the lowest possible detonation velocity (the critical detonation velocity) D_c is obtained for a critical radius of a charge R_c with the minimum reaction zone length or the critical reaction zone length X_c (and the minimum reaction-time or the critical reaction-time t_c). The meeting points of the two groups of curves (the energy consumption and supply) give the corresponding detonation velocities D_1 and D_c respectively which are stable under respective conditions.

§. 2-2 Theoretical formulas

In the present theory no discrimation between the so-called thin wall and the thick wall is necessary. In Fig. 2. case (1) shows a bare charge with reaction time t_0 (reactionzone-length X_0). The radius of a bare charge is R. The case (2) shows how a thin wall elongates the reaction-time and the reaction zone by an amount t_w and X_w respectively.

The value of t_w may be assumed to be as follows.:

$$t_w = \frac{Y}{v_w} \qquad (2\text{-}11)$$

where: Y=thickness of wall
v_w=yelocity of shock wave within wall.

(2-11) is based on the assumption that (1) an effective movement of a wall occurs only at E the point of maximum heat development, that is, at the end of a reaction zone, (2) the wall does not move until a strong shock wave developed at a point E propagates through it into an outer surface, however thin the wall may be.

Because of an extremely rapid action of the detonation even for the thin wall the inertia of the wall does not determine the movement of the wall but the propagation of a shock wave comes first. The reaction-zone-length is elongated by the following amount:

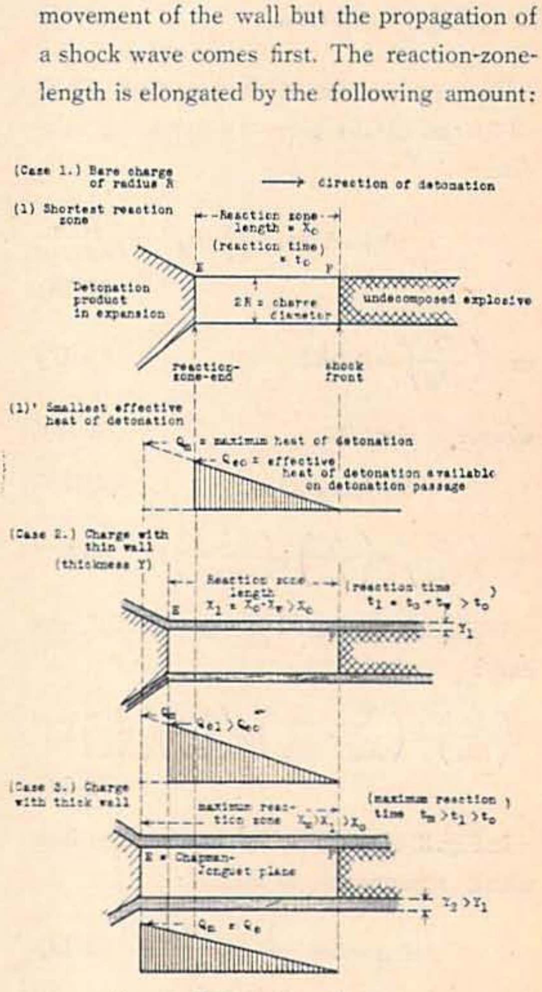

Fig. 2. Effect of confinement

$$X_w = t_w D \qquad (2\text{-}12)$$

However, both t_w and X_w have their upper limits represented by the following equation because the total time of reaction t_t or the total reaction-zone-length X_t can not exceed the maximum reaction-time t_m and the maximum reaction-zone length respectively.

$$t_t = t_0 + t_w = t_0 + \frac{Y}{v_w} \leqq t_m \qquad (2\text{-}13)$$

or $\quad Y \leqq v_w(t_m - t_0) \qquad (2\text{-}13)'$

$$X_t = X_0 + X_w = X_0 + \frac{Y}{v_w} D \leqq X_m \qquad (2\text{-}14)$$

or $\quad Y \leqq \frac{v_w}{D}(X_m - X_0) \qquad (2\text{-}15)$

The combination of the equations (2-2) (2-7) and (2-13) gives the following relation.

$$\frac{D}{D_m} = \sqrt{N} = \sqrt{\frac{t_t}{t_m}} = \sqrt{\frac{t_0 + Y/v_w}{t_m}} \qquad (2\text{-}16)$$

or $\quad \left(\frac{D}{D_m}\right)^2 = A + BY \qquad (2\text{-}17)$

where: $\quad A = t_0/t_m \qquad (2\text{-}17)'$

$$B = 1/(v_w t_m) \qquad (2\text{-}17)''$$

$$\left(\frac{D}{D_m}\right)^2 = \left(\frac{t_0}{t_m}\right) + \left(\frac{1}{v_w t_m}\right) Y \qquad (2\text{-}17)'''$$

While substituting (2-12) into (2-17)'', we have:

$$\left(\frac{D}{D_m}\right)^2 = \left(\frac{X_0}{X_m} \cdot \frac{D_m}{D_0}\right) + \left(\frac{D_m}{v_w}\right)\left(\frac{1}{X_m}\right) Y \qquad (2\text{-}18)$$

In Fig. 3, $(D/D_m)^2 \sim Y$ gives a straight line whose inclination is as follows:

$$\tan\alpha = B = \frac{1}{v_w t_m} = \frac{D_m}{v_w X_m} \qquad (2\text{-}19)$$

From (2-19) we may evaluate the maximum reaction-time t_m and the maximum reaction-zone-length respectively as follows knowing the values of v_w and D_m.

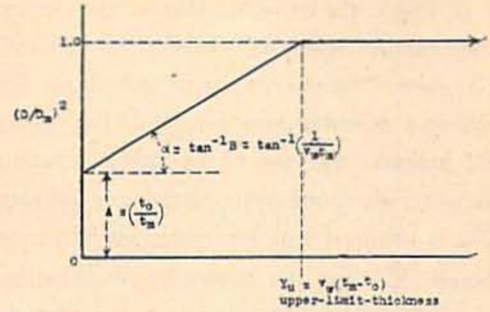

Fig. 3. Effect of thickness of wall on detonation velocity (diameter of explosive constant)

$$t_m = \frac{1}{v_w \tan\alpha} \qquad (2\text{-}19)'$$

$$X_m = \left(\frac{D_m}{v_w}\right) \frac{1}{\tan\alpha} \qquad (2\text{-}19)''$$

In Fig. 3. the value of $A = t_0/t_m$ is easily found, therefore, we may find the reaction-time t_0 of a bare charge as follows:

$$t_0 = A t_m \qquad (2\text{-}20)$$

The reaction-zone-length X_0 for a bare charge is:

$$X_0 = t_0 D_0 \qquad (2\text{-}20)'$$

Above a certain value of

$$Y = Y_u = v_w(t_m - t_0). \qquad (2\text{-}13)'$$

the increase of Y brings about no increase of the detonation velocity D and Y_u may be defined as "the upper-limit-thickness." Above this point we enter the so-called "thick wall" in classical theory, while below Y_u we are dealing with the so-called "thin wall" although in the present theory both cases are covered by one equation.

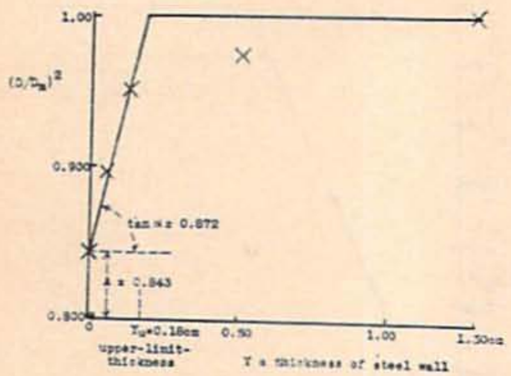

Fig. 4. 60/40 Amatol in a steel casing (R=2.53 cm) (Data form Copp & Ubbelohde)

§. 2-3. Comparison with experimental data of Copp and Ubbelohde.

In Fig. 4. experimental data on 60/40 Ammatol (TNT 40%, coarse NH_4NO_3 60%) due to Copp and Ubbelohde[2] with the radius of explosive $R=2.53$ cm in a steel casing have been plotted according to (2-17). From Fig. 4. we find:

$$\tan\alpha=0.872$$

while $$D_m=6080 \text{ m/sec.}$$

$$v_w=5940 \text{ m/sec.}$$

$$X_m=\left(\frac{6080}{5940}\right)\frac{1}{0.872}=1.17 \text{ cm.}$$

$$t_m=\frac{1.17}{0.608\times10^6}\text{sec}=1.93\mu\text{ sec.}$$

Also from Fig. 4.

$$A=t_0/t_m=0.843 \text{ therefore}$$

$$t_0=0.843\,t_m$$

$$=0.843\times1.93=1.63\,\mu\text{ sec.}$$

$$X_0=1.63\times10^{-6}\times0.558\times10^6=0.91 \text{ cm}$$

$$Y_u=v_w(t_m-t_0)=0.594(1.93-1.63)$$

$$=0.178 \text{ cm.}$$

From Fig. 4. Y_u is found to be 0.18 cm in a good agreement with the calculated value. From Copp's data. $Y_u \doteqdot 0.2$ cm.

Copp & Ubbelohde report that $t=3.31\,\mu$ sec. by use of the equation of Jones that is, the case of "thick wall."

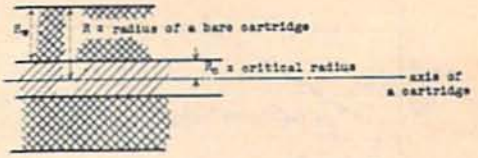

Fig. 5. Structure of bare charges whose radius is bigger than theoretical radius

§. 3. Theory on the effect of the radius *R* of charges

§. 3-1. Assumption

Let us assume that there exists a critical radius R_c of a cartridge below which no stable detonation occurs. At a radius of R_c the detonation velocity is the critical detonation velocity D_c. At this critical radius the reaction time t_c is the shortest possible one. As the radius of cartridges increases bigger than R_c as is illustrated in Fig. 5, the reaction time t increases by the amount t_w due to the wall effect of the part of a cartridge R_w due to the wall effect of the part of a cartridge R_w, that is, we may assume, as in §. 2., the outer part of a cartridge acts as a wall around "a charge of radius R_c (critical radius)," because the detonation velocity D for R is determined by D in the axial core where D attains the highest value on the section of a cartridge. Then the similar argument as in §. 2. leads to the following expression.

$$\frac{D}{D_m}=\sqrt{\frac{t_c+R_w/v_e}{t_m}} \qquad (3\text{-}1)$$

where

v_e=velocity of the sound wave in detonation products.

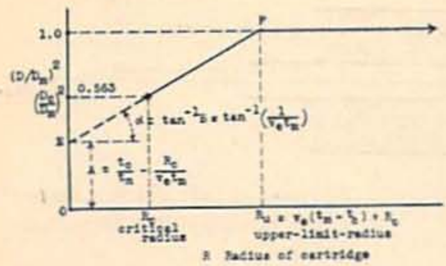

Fig. 6. Effect of radius of bare cartridges on detonation velocity

$$v_e \doteqdot \frac{v_2}{v_1} D \doteqdot \frac{5}{6} D \qquad (3\text{-}2)$$

where v_2 and v_1 are the specific volumes of detonation products and the original explosive respectively. To a first approximation v_e is assumed to be a constant,

$$R_w = R - R_c \qquad (3\text{-}3)$$

where

R_w=thickness of an outer shell of a cartridge around a critical radius

R=radius of (a total) cartridge

R_c=critical radius

D=detonation velocity for a radius R

D_m=maximum (or ideal or theoretical) detonation velocity

t_m=maximum reaction time

t_c=critical (or minimum) reaction time

$$(D/D_m)^2 = (t_c/t_m) + (R - R_c)/(v_e t_m) \qquad (3\text{-}4)$$

or $$(D/D_m)^2 = [t_c/t_m - R_c/(v_e t_m)] + [1/(v_e t_m)]R \qquad (3\text{-}5)$$

or $$(D/D_m)^2 = A + BR \qquad (3\text{-}5)'$$

where $$A = t_c/t_m - R_c/(v_e t_m) \qquad (3\text{-}5)''$$

$$B = 1/(v_e t_m) \qquad (3\text{-}5)'''$$

The equation (3-5) is represented as a straight line with respect to $(D/D_m)^2$ and R in Fig. 6. The values of A and B in (3-5)' may be easily found in Fig. 6. Therefore,

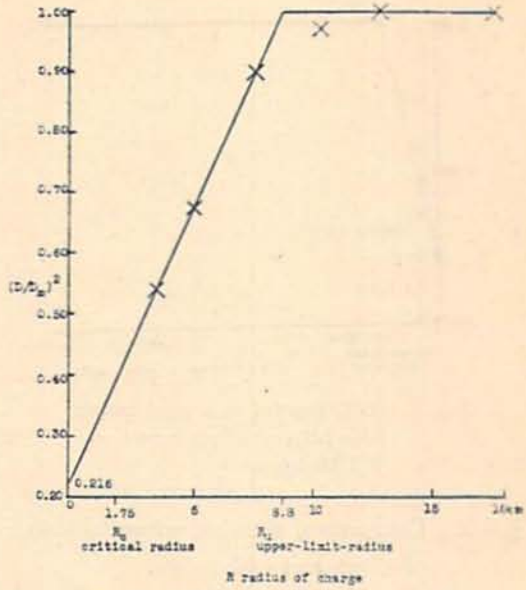

Fig. 7. 50/50 Amatol (bare charge) (Data from Cook et al.)

$$A = \frac{1}{t_m}\left(t_c - \frac{R_c}{v_e}\right), \quad \left(At_m = t_c - \frac{R_c}{v_e}\right)$$

or for the critical reaction time:

$$t_c = At_m + \frac{R_c}{v_e} \qquad (3\text{-}6)$$

$$t_m = \frac{1}{v_e \tan\alpha} = \frac{1}{v_e B} \qquad (3\text{-}7)$$

$$X_m = D_m t_m \qquad (3\text{-}8)$$

$$X = D_c t_c + \left(\frac{R_w}{v_e}\right) D \qquad (3\text{-}9)$$

$$t = t_c + \frac{R_w}{v_e} \qquad (3\text{-}9)'$$

As the total reaction time t cannot exceeds the maximum reaction time t_m we find:

$$t_t = t_c + \frac{R_w}{v_e} \geqq t_m \text{ or } R_w \geqq v_e(t_m - t_c) \qquad (3\text{-}10)$$

Above a certain radius R_u the detonation velocity D does not increase when R increases. The value of R_u is given by:

$$R_u = (R_w)_{max.} + R_c = v_e(t_m - t_c) + R_c \qquad (3\text{-}10)'$$

where R_u=upper-limit-radius.

§. 3-3. Comparison with experimental data of Cook et al.

3-3-(1) 50/50 Amatol

Fig. 7. shows the results[4] on bare charges of loading density

$$d=1.53\ \mathrm{g/cm^3}.$$

$$\tan\alpha=0.0891$$

$$v_e=\frac{5}{6}D_m=\frac{5}{6}\times 0.6351\times 10^6$$

$$=0.529\times 10^6\mathrm{cm/sec}.$$

$$t_m=\frac{1}{v_e\tan\alpha}=21.2\mu\ \mathrm{sec}.$$

$$X_m=D_m t_m=0.6351\times 21.2=13.48\mathrm{cm}$$

$$A=0.216$$

$$R_c=1.75\mathrm{cm}$$

$$t_c=At_m+\frac{R_c}{v_e}=0.216\times 21.2\times 10^{-6}$$

$$+\frac{1.75}{0.529}\times 10^{-6}=7.57\mu\mathrm{sec}.$$

critical velocity$=D_c=4695$m/sec.

$$X_c=t_cD_c=7.57\times 10^{-6}\times 0.4695\times 10^6$$

$$=3.55\mathrm{cm}.$$

$$R_u=v_e(t_m-t_c)+R_c$$

$$=0.529\times 10^6(21.2-7.57)\times 10^{-6}+1.75$$

$$=7.21+1.75=8.96\mathrm{cm}$$

From Fig. 7. $R_u=8.8$cm.

3-3-(2) 2-4-Dinitrotoluene.

$$d=0.95\mathrm{g/cm^3}$$

Experimental data have been read from a smoothed experimental curve $D\sim R$.[10]

Fig. 8. shows the result. The calculated values of t_m etc. are summarized in Table 2.

3-3-(3) TNT[6]-1 $d=1.0\mathrm{g/cm^3}$

Fig. 9. shows the result. The calculated values are shown in Table 2.

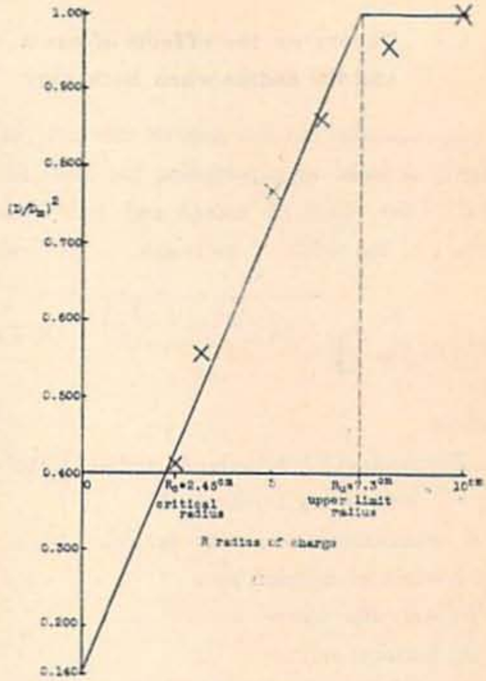

Fig. 8. 2-4-DNT (bare charge)
(Data from Cook et al.)

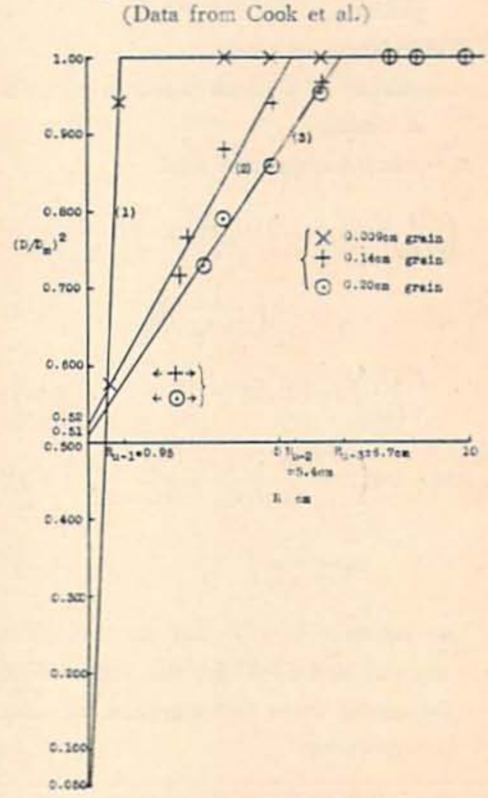

Fig. 9. TNT (bare charge)
(Data from Cook et al.)

§.4. Theory on the effects of cases and the radius when both vary

An equation for this general case may be easily deduced by superposing the cases for §.2. (the effect of cases) and §.3. (the effect of the radius of cartridges) as follows:

$$\frac{D}{D_m}=\sqrt{\frac{t_c+\left(\frac{R_w}{v_e}\right)+\left(\frac{Y}{v_w}\right)}{t_m}} \quad (4\text{-}1)$$

where

D=detonation velocity at radius R and with casing thickness Y

D_m=maximum detonation velocity

t_c=critical reaction time

$R_w=R-R_c$ where

R_c=critical radius

v_e=velocity of sound wave in detonation products

Y=thickness of cases

v_w=velocity of a shock wave in material of casing

t_m=maximum reaction time

$$\left(\frac{D}{D_m}\right)^2=\left\{\left(t_c-\frac{R_c}{v_e}\right)\frac{1}{t_m}+\frac{Y}{v_w}\frac{1}{t_m}\right\}+\left(\frac{1}{v_e t_m}\right)R \quad (4\text{-}2)$$

or $$\left(\frac{D}{D_m}\right)^2=A+BR \quad (4\text{-}3)$$

where: $$A=\left(t_c-\frac{R_c}{v_e}\right)\frac{1}{t_m}+\frac{Y}{v_w}\frac{1}{t_m} \quad (4\text{-}3)'$$

$$B=\frac{1}{v_e t_m} \quad (4\text{-}3)''$$

The equation (2-17)''' for the wall-effect and the equation (3-5) for the radius-effect are the special cases of the general equation (4-2) respectively.

10) M. A. Cook & W. S. Partridge: J. Phy. Chem. 56. 673-675 (1955).

§.5. Application of the theory

§.5-1. Principle

One of the applications of the theory presented in this paper is to find the relations between the detonation velocity D (and the detonation pressure P_D which is of practical importance from the standpoint of the theory and practice of blasting. P_D is easily calculated, to a first approximation, when D is known) and the thickness of cases Y or the diameter of charges R from experiments as few as possible.

For example let us consider bare charges.

(1) Find a detonation velocity D_1 for radius R_1 and D_2 for R_2.

(2) Calculate the maximum detonation velocity D_m or find it by experiments for a much larger radius R_m.

(3) Plot $(D_1/D_m)^2$ and $(D^2/D_m)^2$ on y-axis, R_1 and R_2 on the x-axis as shown in Fig. 6. From the straight line EF find the numerical values of α, and R_u.

$$\tan\alpha=B=\frac{1}{v_e t_m} \quad (5\text{-}1)$$

$$A=\frac{t_c}{t_m}-\frac{R_c}{v_e t_m} \quad (5\text{-}2)$$

$$R_u=v_e(t_m-t_c)+R_c \quad (5\text{-}3)$$

(4) Calculate the velocity of sound in detonation products thermodynamically or to a first approximation by the following equation.

$$v_e=\frac{5}{6}D_m \quad (5\text{-}4)$$

(5) $$\frac{D_c}{D_m}=0.75 \text{ (Cook)} \quad (5\text{-}5)$$

or $$\left(\frac{D_c}{D_m}\right)^2=0.563 \quad (5\text{-}5)'$$

In other words it may be stated that "the critical radius of explosive charges is the radius at which a cartridge detonates with an effective heat of detonation which is about one half of the maximum heat of detontion."

Then from Fig. 6. we can find the critical radius R_c.

(6) The critical reaction time t_c is found as follows:

From (5-2),

$$t_c = At_m + \frac{R_c}{v_e} \qquad (5\text{-}6)$$

From (5-3),

$$t_c = t_m - \frac{R_u - R_c}{v_e} = t_m - \frac{R_{wu}}{v_e} \qquad (5\text{-}7)$$

where R_{wu}=upper-limit-thickness of an explosive shell around a critical radius

(7) The maximum reaction-zone-length:

$$X_m = D_m t_m \qquad (5\text{-}8)$$

The critical reaction-zone-length:

$$X_c = D_c t_c \qquad (5\text{-}9)$$

By use of the procedure described above we may find the important characteristics of explosive cartridges, by the minimum number of detonation velocity measurements, which have great practical and theoretical importance.

§. 5-2. Example

In an ammonium nitrate permitted explosive, whose loading density d=1.0g/cm³, the experimental data on R and D have been found as shown in Table. 1. In this case D_m may be supposed to be:

$$D_m \fallingdotseq 3620 \text{m/sec.}$$

Table 1. Ammon explosive

R cm	0.85	1.25	1.6	2.0
D m/sec	2870	3240	3420	3620

From Fig. 10. we find:

Table 2. Calculated characteristics of reaction zones in detonation of solid explosives

Explosives (charge / bare)	The present theory						Previous theories				
							t μsec.			X cm	
	t_m / t_c	X_m / X_c	R_u / R_c	X_m / R_u	R_c / X_c	D_c / D_m *	nozzle (Jones)	curved front (Eyring)	det. head (Cook)	nozzle	curved front
50/50-Amatol	21.2/7.57 = 2.8 μsec	13.48/3.55 = 3.80 cm	8.96/1.75 = 5.12 cm	1.50	2.03	(0.740)	3.4	1.7	20 (28.5)	(1.5)	(0.75)
2-4-DNT	26.2/11.23 = 2.33	6.75/2.8 = 2.41	7.31/2.45 = 2.98	0.92	1.14	(0.642)				2.9	1.5
(1) TNT0.009cm	2.38/1.54 = 1.545	1.204/0.594 = 2.03	0.954/0.6 = 1.59	1.16	0.99	(0.761)				0.0936	0.0432
(2) TNT 0.14cm	26.74/18.89 = 1.415	13.54/7.35 = 1.84	5.4/2.1 = 2.57	2.51	3.50	(0.769)				1.456	0.672
(3) TNT 0.20cm	32.5/22.39 = 1.45	16.45/8.46 = 1.945	6.70/2.45 = 2.74	2.45	3.45	(0.748)				2.08	0.96
Permitted ammon explosive	10.75/6.08 = 1.77	3.89/1.66 = 2.34	2.00/0.59 = 3.39	1.95	2.81	0.75⊙	1.5～2.8 △	1.5～1.9		8.0～5.4 △	0.53
Theoretical values	1.78	2.36				(0.75)⊙					

Note * experimental data

⊙ assumed values

△ as the straight line[3] is not obtained for $(D_m/D)^2 \sim (1/R)^2$ (Jones-Taylor) t and X have various values.

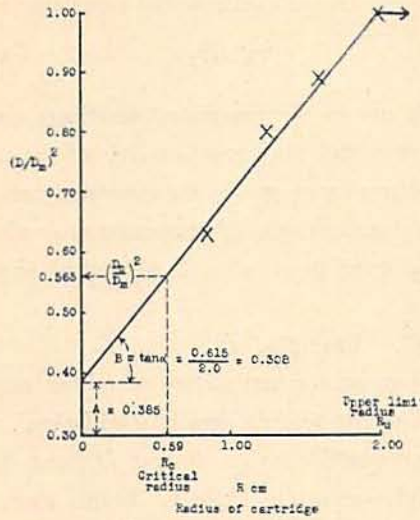

Fig. 10. Permitted Ammon explosive $d=1.0\text{g/cm}^3$ (bare charge)

$R_c=0.59\text{cm}$

$A=0.385$

$B=\tan\alpha=0.308$

Then: $v_s=0.302\times10^6\text{cm/sec.}$

$t_m=10.75\mu\text{sec.}$

$D_c=0.272\times10^6\text{cm/sec.}$

$X_m=3.89\text{cm}$

(5-6) $t_c=6.09\mu\text{sec.}$

(5-7) $t_c=6.08\mu\text{sec.}$ $X_c=1.66\text{cm}$

§. 6. General rules on the reaction time ratio t_m/t_c and the reaction zone length ratio X_m/X_c.

If we assume the following experimental rule (6-1) among the critical and maximum detonation velocity we may find some rules of practical use for the reaction time and the reaction zone length.

$$D_c/D_m=0.75 \qquad (6\text{-}1)$$

(1) Reaction time ratio

The ratio of the maximum reaction time t_m to the minimum (critical) reaction time t_c is as follows:

$$\frac{t_m}{t_c}=\left(\frac{D_m}{D_c}\right)^2=\frac{1}{(0.75)^2}=1.78 \quad (6\text{-}2)$$

(2) Reaction zone length ratio

The ratio of the maximum reaction zone length X_m to the critical (minimum) reaction zone length X_c is as follows:

$$\frac{X_m}{X_c}=\frac{D_m t_m}{D_c t_c}=\left(\frac{D_m}{D_c}\right)\left(\frac{D_m}{D_c}\right)^2$$
$$=\left(\frac{D_m}{D_c}\right)^3=\left(\frac{1}{0.75}\right)^3=2.38 \qquad (6\text{-}3)$$

In Table 2. the results of the calculations based on the present theory have been summarized in comparison with the values of the constant reaction time t and the constant reaction zone length calculated by the classical equations such as Jones and Eyring.

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