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A NEW SOLUTION OF THE INTERIOR BALLISTICS OF GUNS

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I. General

1) Introduction

E. Sarrau of France and others have tried to obtain a general solution of the Interior Ballistics theoretically but failed until P. Charbonnier[1] was the first to achieve a sucess. However, in his method of the solution, he eliminated the second term and below in expanding into a series on the integration of the equation relating to the fraction of weight burnt at time t, z and expansion ratio θ. Thus quite an error cannot be avoided.

G. Sugot[2] improved the above and calculated to the second term. Instead of taking the co-volume η as a constant, he used η_z as a function of z and obtained a more accurate solution. However, as this method is complicated, and in practical calculation many tables are necessary within a certain range, in special cases it is still quite erroneous.

The present author has used the fundamental equations used by Charbonnier-Sugot, and first obtained a relation of the velocity of projectile v with the fraction of weight burnt z, and from this obtained the solution of expansion ratio θ as a function of velocity v. In the same manner, the relation of the pressure P with v has been obtained. Thus, by these methods, equations with (v, z), (θ, v) and (P, v) can be obtained without abbreviating any of the terms. The author has already made a report[3] using η_z as a constant, but in this paper he will discuss about the accurate solution by using

$$\eta_z = \frac{1}{\delta} + \left(\eta - \frac{1}{\delta}\right)z \qquad (1)$$

2) Theoretical Form Function of Propellants Burning

The theoretical form function $\varphi(z)$ can be reduced from the geometrical form of a grain of propellant. P. Charbonnier has proved $\varphi(z) = \sqrt{1-Gz}$ for cylindrical grain and $(1-z)^0$ for tubular and ribbon grain, z being the fraction of the weight of the powder grain burnt at time t. Our experiments and calculations have shown us that[4], they verified that the theoretical form fnnctions of prismatic, cylindrical, tubular ribbon and multitubular grains could be satisfactorily expressed by the formula,

$$\varphi(z) = \sqrt{1-Gz} \qquad (2)$$

In this case, when $G=1$ it is a cylindrical grain, and $G=0$ when a grain having constant burning surface and even in American seven perforated multitubular powder the greater parts of combustion follow $\sqrt{1+G'z}$, taking $G'=-G$. Previously, the author[5] reported that the rate of brning of smokeless powder is proportional to pressure. Therefore, the relation between burning velocity, pressure and theoretical form function can be expressed by

$$\frac{dz}{dt} = AP\varphi(z) \qquad (3)$$

3) Co-volume correction

In my previous paper[3], the co-volume η during the burning of propellant was treated as a constant, but here η_z as a variable of z as shown in equation (1) same as Sugot did in his internal ballistics[2]. Therefore, taking some mean value η' as a constant between $\eta = 1/\delta$ and $\eta_z = \eta$ corresponding to $z = 0$ and $z = 1$, we have

$$c - \eta_z\omega = (c - \eta'\omega)\{1 + b\theta'(1 - kz)\} \qquad (4)$$

where

$$\left.\begin{aligned} b &= \left(\eta' - \frac{1}{\delta}\right) \Big/ \left(\frac{1}{\Delta} - \eta'\right) \\ \theta' &= \left(\frac{1}{\Delta} - \eta'\right)\left(\frac{\rho}{\Delta} - \eta'\right) \\ &= (c_0 - \eta'\omega)/(C - \eta'\omega) \\ k &= \left(\eta - \frac{1}{\delta}\right) \Big/ \left(\eta' - \frac{1}{\delta}\right) \\ \rho &= C/c_0 \end{aligned}\right\} \qquad (5)$$

If k is near to 1, θ' near to θ, and assuming the density of powder $\delta = 1.56$, and $\eta = 1$, then $\eta' = 0.82$. We find for $\Delta = 0.2$, b=0.047 Even the highest density of loading $\Delta = 0.6$, $b = 0.17$ which is a much smaller figure than 1, therefore term b^2 can be eliminated.

4) Notations and Fundamental Equations

i) Notations

All notations are nearly the same as those used by Carbonnier-Sugot. Kg, dm, and sec. are used as units.

ω	Weigh of Powder charge
p	Weight of projectile
z	Fraction of weight burnt at time t
f	Force constant of explosive
A	Vivacity
i	Coefficient of interior ballistics
η	Co-volume
γ	Index of adiabatic expansion or Specific heats ratio
P_0	Shot start pressure
P	Pressure of propellant gas
P'	Maximum pressure in closed vessel
P_m	Maximum pressure
P_E	Muzzle pressure
x	Length of travel of projectile
a	Mean diameter of bore or calibre
$\sigma = \pi a^2/4$	Sectional area of the bore
c	Volume of the bore
C	Total volume of the bore
c_0	Volume of powder chamber
$\rho = C/c_0$	
$\Delta = \omega/c_0$	Loading density
v	Velocity of projectile
V	Muzzle velocity of projectile
t	Time

ii) Auxiliary equations

ii—1 $P = \frac{ip}{g}\left(1 + \frac{\omega}{2p}\right)$

ii—2 $P' = f \cdot \Delta/(1 - \Delta\eta)$

ii—3 $z_0 = P_0/P'$

ii—4 $\varphi(z) = \sqrt{1 - Gz}$

ii—5 $\sigma x = c - c_0$

ii—6 $\theta = \frac{(c_0 - \eta\omega)}{(c - \eta\omega)}$

ii—7 $\Theta = \frac{(c_0 - \eta'\omega)}{(C - \eta\omega)}$

iii) Auxiliary notations

iii—1 $K = \frac{A\mu}{2\sigma}$

iii—2 $L = K^2G + \frac{\gamma-1}{2}\frac{\mu}{f\omega}$

iii—3 $M = \frac{K}{L}\varphi(z_0) = M_0\varphi(z_0)$

iii—4 $N = \frac{z_0}{L}$

iii—5 $R = \sqrt{1+\frac{N}{M^2}}$

iii—6 $\alpha = \frac{f\omega L}{\mu}$

iii—7 $\beta = \frac{2\alpha}{1+2\alpha}$

iii—8 $s = N + 2Mv - v^2$

iv) Fundamental equations of interior ballistics

(I) Résal's Equation

$$P(c-\eta_z\omega) + \frac{\gamma-1}{2}\mu v^2 = f\omega z$$

(II) Newton's equation of motion

$$\mu\frac{dv}{dt} = \sigma P$$

(III) Geometrical form function of propellant grain

Taking the burning velocity as proportional to P,

$$\frac{dz}{dt} = AP\varphi(z) = AP\sqrt{1-Gz}$$

(IV) Equation of expansion of propellant gas when all burnt

$$P(c-\eta\omega)^{\gamma} = \text{const.}$$

II. Methods of Solutions

5) During burning of powder grains

i) The equation of velocity v and z fraction of weight of powder burnt at time t.

From the fundamental equations (II) and (III), the following relations will be obtained,

$$\frac{dz}{dt} = A\frac{\mu}{\sigma}\frac{dv}{dt}\varphi(z)$$

If v is integrated from o to v, and z from z_0 to z

$$A\frac{\mu}{\sigma}v = \int_{z_0}^{z}\frac{dz}{\varphi(z)}$$

$$= \frac{2}{G}[\sqrt{1-Gz_0} - \sqrt{1-Gz}] \qquad (6_1)$$

This gives z as a function of v.

$$z = z_0 + 2K\varphi(z_0)v - K^2Gv^2 \qquad (6)$$

ii) Equation of velocity v in relation to volume of bore c

Substitute z of equation (6) in the fundamental equation (I), and at the same time by inserting the fundamental equation (II) for P, and we get

$$\frac{\mu}{\sigma}v\frac{dv}{dx}(c-\eta_z\omega) + \frac{\gamma-1}{2}\mu v^2$$
$$= f\omega(z_0 + 2K\varphi(z_0)v - K^2Gv^2)$$

By inserting $\sigma dx = dc$ from the auxiliary notation (ii—5)

$$\frac{c-\eta_z\omega}{dc}vdv = \frac{f\omega}{\mu}\left[z_0 + 2K\varphi(z_0)v - \left(K^2G + \frac{\gamma-1}{2}\frac{\mu}{f\omega}\right)v^2\right] \qquad (7)$$

To use auxiliary notations (iii—1, 2, 3 and 6)

$$\frac{\alpha\cdot dc}{c-\eta_z\omega} = \frac{vdv}{N+2Mv-v^2} \qquad (8)$$

rewriting η_z with equation (4)

$$\frac{\alpha dc}{c-\eta'\omega} = \frac{[1+b\theta'(1-kz)]vdv}{N+2Mv-v^2} \qquad (9)$$

also from θ' of equation (5)

$$-d\theta' = \frac{(c_0-\eta\omega)dc}{(c-\eta'\omega)^2} = \frac{\theta' dc}{c-\eta'\omega}$$

and substituting this in equation (9)

$$-\alpha\frac{d\theta'}{\theta'}=\frac{[1+b\theta'(1-kz)]\,v\,dv}{N+2Mv-v^2} \tag{10}$$

As $y=\dfrac{1}{\theta'}$ or, $dy=\dfrac{-d\theta'}{\theta'^2}$

$$\alpha\frac{1}{y}\frac{dy}{dv}-\frac{v}{N+2Mv-v^2}$$
$$-b\frac{1}{y}(1-kz)\frac{v}{N+2Mv-v^2}=0$$

Insert equation (6) in z and let $F_1(v)$ and $F_2(v)$ be

$$F_1(v)=\frac{v}{\alpha(N+2Mv-v^2)} \tag{11}$$

$$F_2(v)=\frac{v}{\alpha}\times\frac{\{kK^2Gv^2-2kK\varphi(z_0)v+(1-kz_0)\}}{N+2Mv-v^2} \tag{12}$$

and the following differential equation will be obtained

$$\frac{dy}{dv}+yF_1(v)-bF_2(v)=0 \tag{13}$$

The solution of the above equation is:

$$y=\exp.\left(-\int F_1(v)\,dv\right)\Big[b\int F_2(v)$$
$$\times\exp.\left(\int F_1(v)\,dv\right)dv+\text{const.}\Big] \tag{14}$$

If the limits of integration are $v=0$, $\theta'=1$ then const. $=1$. Next, the integration of equation (11) is

$$-\int_0^v F_1(v)\,dv=\frac{1}{2\alpha}\log\left(\frac{N}{s}\right)$$
$$+\frac{1}{2\alpha R}\log\left[\frac{M(R-1)+v}{M(R+1)-v}\,\frac{R+1}{R-1}\right]$$

Consequently

$$\exp.\left(-\int F_1(v)\,dv\right)=\left(\frac{N}{s}\right)^{\frac{1}{2\alpha}}.$$
$$\left[\frac{M(R-1)+v}{M(R+1)-\gamma}\,\frac{R+1}{R-1}\right]^{\frac{1}{2\alpha R}} \tag{15}$$

In the next place, by equation (6), let

$$1-kz=kK^2Gv^2-2kK\varphi(z_0)v$$
$$+1-kz_0=a'v^2-b'v+c' \tag{16}$$

by substituting this in equation (12), we have

$$F_2(v)\exp.\left[\int F_1(v)\,dv\right]=\frac{v}{\alpha}$$
$$\frac{a'v^2-b'v+c'}{s}\left(\frac{s}{N}\right)^{\frac{1}{2\alpha}}$$
$$\times\left(\frac{M(R+1)-v}{M(R-1)+v}\,\frac{R-1}{R+1}\right)^{\frac{1}{2\alpha R}} \tag{17}$$

It is difficult to integrate this equation after substituting in equation (14), but as the term of $F_2(v)$ is small containing b as a factor, it can reasonably be treated just as in the case of taking $\varphi(z_0)=1$ namely $z_0=0$ in the solution of the equation (17). This proof will be explained in the next section.

In this case, as equation (6) leads to

$$z=Kv(2-KGv) \tag{18}$$

And by the auxiliary notation iii—3, iii—4 $N=0$ and $M=M_0$. Then $F_1(v)$, and $F_2(v)$ of equations (11) and (12) lead to, taking $2kK=b_0'$

$$F_1(v)=\frac{-1}{\alpha(2M_0-v)}$$

$$F_2(v)=\frac{a'v^2-b'_0v+1}{\alpha(2M_0-v)}$$

$$F_2(v)\exp.\left(\int F_1(v)\,dv\right)=\alpha^{-1}(2M_0)^{-\frac{1}{\alpha}}.$$
$$\times[a'v^2-b'_0v+1](2M_0-v)^{\frac{1}{\alpha}-1} \tag{19}$$

Next let $2M_0-v=X$ and we have $dv=-dX$ and let $\dfrac{1}{\alpha}-1=\varepsilon$, therefore

$$-\int_{2M_0}^{2M_0-v}F_2(v)\exp.\left(\int F_1(v)\,dv\right)dv$$
$$=\frac{1}{\alpha}\left\{\left(1-\frac{v}{2M_0}\right)^{\varepsilon+1}\left[\frac{4a'M_0^2}{\varepsilon+3}\left(1-\frac{v}{2M_0}\right)^2\right.\right.$$

$$+\frac{b'_0-4a'M_0}{\varepsilon+2}2M_0\left(1-\frac{v}{2M_0}\right)$$
$$+\frac{4a'M_0^2-2b'_0M_0+1}{\varepsilon+1}\Bigg]$$
$$-\Bigg[\frac{4a'M_0^2}{\varepsilon+3}+\frac{b'_0-4a'M_0}{\varepsilon+2}2M_0$$
$$+\frac{4a'M_0^2-2b'_0M_0+1}{\varepsilon+1}\Bigg]\Bigg\}$$
$$=E(M_0,v) \qquad (20)$$

In that case y of equation (14) gives

$$y=\frac{1}{\theta'}=\left(\frac{N}{s}\right)^{\frac{1}{2\alpha}}\left(\frac{M(R-1)+v}{M(R+1)-v}\times\frac{R+1}{R-1}\right)^{\frac{1}{2\alpha R}}[1-bE(M_0,v)] \qquad (21)$$

This is the equation to which the corrected term b has been added.

iii) The relation between pressure P, velocity v, and volume of bore c.

From the fundamental equation (I) and (II)

$$\mu v\frac{dv}{dx}=\sigma P=\sigma\frac{f\omega z-\frac{\gamma-1}{2}\mu v^2}{c-\eta_z\omega}$$

Substitute equation (6) for z and by using auxiliary notations,

$$P(c-\eta_z\omega)=f\omega L(N+2Mv-v^2)$$

Introduce equation (4) for $c-\eta_z\omega$, equa- (16) for z, and equation (5) for θ', and we have

$$P=P'\theta'L\frac{N+2Mv-v^2}{1+b\theta'(a'v^2-b'v+c')} \qquad (22)$$

Thus P can be found as a function of v and θ'.

6) The point of just burnt

At the point of just burnt $z=1$, $v=v_1$, we have from equation (6_1)

$$v_1=\frac{(\sqrt{1-Gz_0}-\sqrt{1-G})}{KG} \qquad (23)$$

Insert equation (21) in the above

$$\frac{1}{\theta'_1}=\left(\frac{N}{s}\right)^{\frac{1}{2\alpha}}\left[\frac{M(R-1)+v_1}{M(R+1)-v_1}\frac{R+1}{R-1}\right]^{\frac{1}{2\alpha R}}\times[1-bE(M_0,v_1)] \qquad (24)$$

and in the same way from equation (22) pressure P_1 at the point of just burnt becomes

$$P_1=P'\theta'_1L\frac{s_1}{1+b\theta_1'(a'v_1^2-b'v_1+c')} \qquad (25)$$

7) Determination of all burnt or not all burnt in the bore

The condition of all burnt or not at the muzzle can be decided by $\theta\gtreqless\Theta$ on inserting the conditional equation (23) in equation (21), as fallows

$$\Theta^{2\alpha}\gtreqless\left(\frac{s_1}{N}\right)\left(\frac{M(R+1)-v_1}{M(R-1)+v_1}\frac{R-1}{R+1}\right)^{\frac{1}{R}}\times[1-bE(M,v_1)]^{-2\alpha} \qquad (26)$$

Initial velocity V and muzzle pressure P_E can be found from equation (22), and Z from equation (16).

8) Initial velocity V and the muzzle pressure P_E when the charge is not all burnt in the bore

When the powder is not all burnt before the shot reaches muzzle, initial velocity V can be obtained from equation (24) taking $\theta=\Theta$, and muzzle pressure P_E from equation (6_1)

$$V=\frac{\sqrt{1-Gz_0}-\sqrt{1-Gz}}{KG}$$

9) Solution after all burnt

An analytical solution of P, v and c after all burnt can be treated by the ordinary method using the fundamental equation IV. The initial condition being P_1, c_1, and θ_1 at the point of just burnt of charge.

$$P = P_1\left(\frac{c_1-\eta\omega}{c-\eta\omega}\right)^\gamma = P_1\left(\frac{\theta}{\theta_1}\right)^\gamma \quad (27)$$

where P_1 can be found from equation (25) so the relation of pressure and position can be obtained. Concerning velocity v, using the fundamental equation (II) and inserting equation (27) in P we have

$$\mu v dv = \sigma P dx = P_1\left(\frac{c_1-\eta\omega}{c-\eta\omega}\right)^\gamma dc$$

Integrating this from v_1 to v, and from c_1 to c, and inserting equation (25) for P, the velocity after all burnt can be obtained as follows,

$$v^2 = v_1^2 + \frac{2\alpha}{\gamma-1}\left[1-\left(\frac{\theta}{\theta_1}\right)^{\gamma-1}\right] \times \frac{s_1}{1+b\theta'(a'v_1^2-b'v_1+c)} \quad (28)$$

Finally, at the muzzle, pressure P and expansion θ of equation (27) should be made P_B and Θ, and the initial velocity V is

$$V^2 = v_1^2 + \frac{2\alpha}{\gamma-1}\left[1-\left(\frac{\Theta}{\theta_1}\right)^{\gamma-1}\right] \times \frac{s_1}{1+b\theta'(a'v_1^2-b'v_1+c)} \quad (28_1)$$

10) Maximum pressure

By differentiating equation (22), we obtain a condition of maximum pressure putting $\frac{dP}{dv}=0$

$$\frac{1}{P}\frac{dP}{dv} = \frac{1}{\theta'}\frac{d\theta'}{dv} + \frac{2(M-v)}{N+2Mv-v^2} - \frac{1}{1+b\theta'(a'v^2-b'v+c')}\left[b\theta'(2a'v-b') + b(a'v^2-b'v+c')\frac{d\theta'}{dv}\right] = 0$$

If we use equation (10) for $\frac{d\theta'}{\theta'}$, θ' can be expressed as a function of v, and the velocity $\bar{v}$ corresponding to maximum pressure can be expressed

$$\bar{v} = \frac{2\alpha}{1+2\alpha}M - \frac{\alpha b\theta'(2a'\bar{v}-b')(N+2M\bar{v}-\bar{v}^2)}{1+b\theta'(a'\bar{v}^2-b'\bar{v}+c')} \quad (29)$$

as the second term on the right hand of this equation contains b, and in order to take the approximate value of v, put $\bar{v}=\frac{2\alpha}{(1+2\alpha)}M_0=\beta M_0$ and in the same way for $\bar{\theta}'$, take $\bar{\theta}=\left(1-\frac{\beta}{2}\right)^{\frac{1}{\alpha}}$. Inserting this as the first approximation from equation (29)

$$\bar{v}=\beta M - b\beta\left(1-\frac{\beta}{2}\right)^{\frac{1}{\alpha}} \times \frac{(2a'\beta M-b')(N+(2-\beta)\beta M^2)}{1+b(1-\frac{\beta}{2})^{\frac{1}{\alpha}}(a'\beta^2M^2-b'\beta M+c')} \quad (30)$$

Thus, $\bar{v}$ can be represented quite accurately by the second approximation.

11) On the justification of the lower limit integration of the differential equation (13)[6]

To prove what error may occur by assuming $z_0=o$, let exact equations (11) and (12) be $\bar{F}_1(V)$ and $\bar{F}_2(V)$ respectively. Then

$$\bar{F}_1(v) = \frac{-v}{\alpha s};\quad \bar{F}_2(v) = \frac{v(av^2-b'v+c')}{\alpha s}$$

Next let $F_1(v)$ and $F_2(v)$ be approximate equations, and we have

$$F_1(v) = -\frac{1}{\alpha(2M_0-v)}, \quad F_2(v) = \frac{(a'v^2-b'v+c)}{\alpha(2M_0-v)} \quad (31)$$

Then problem arises as to what the error will be when $\int_0^v \bar{F}_2 \exp.\left(\int_0^v \bar{F}_1 dv\right) dv$ is substituted by $\int_0^v F_2 \exp.\left(\int_0^v F_1\,dv\right) dv$.

Mathematically it is true that when z tends to zero, $\bar{F}_1$ converges to F_1 uniformly and also $\bar{F}_2$ tends to F_2 if $v \geqq \varepsilon > 0$. However, if we choose an interval to reach $v=0$, this convergency does not become uniform and a question arises in the approximation mentioned above. This can be treated as follows, at the vicinity of zero with the positive value of v

$$-F_1 > -\bar{F}_1 > 0 \qquad (32)$$

$$F_2 > \bar{F}_2 > 0 \qquad (33)$$

From the former equation

$$1 > \exp.\left(\int_0^v \bar{F}_1 dv\right) > \exp.\left(\int_0^v F_1 dv\right)$$

therefore

$$F_2 \exp.\left(\int_0^v \bar{F}_1 dv\right) > F_2 \exp.\left(\int_0^v F_1 dv\right)$$

$$\therefore \quad F_2 \exp.\left(\int_0^v F_1 dv\right) - \bar{F}_2 \exp.\left(\int_0^v \bar{F}_1 dv\right) < (F_2-\bar{F}_2) \exp.\left(\int_0^v \bar{F}_1 dv\right) < (F_2-\bar{F}_2)$$

which leads to

$$\int_0^v F_2 \exp.\left(\int_0^v F_1 dv\right) dv - \int_0^v \bar{F}_2 \times \exp.\left(\int_0^v \bar{F}_1 dv\right) dv < \int_0^v (F_2-\bar{F}_2)\, dv \qquad (34)$$

with this equation the error can be ascertained.

$$\alpha\int_0^v (F_2-\bar{F}_2)\, dv =$$

$$2kK[KG(M-M_0)+1-\varphi(z_0))]v - [4kKM_0(KGM_0-1)+1]\cdot ln\frac{2M_0-v}{2M_0}$$
$$+\frac{1}{2}\left\{4kKM[KGM-\varphi(z_0)]+kK^2GN+1-kz_0\right\} ln\frac{s}{N}$$
$$+\frac{M\{[4kKM(KGM-\varphi(z_0)]+kK^2GN+1-kz_0\}+2kKN(KGM-\varphi(z_0))}{2\sqrt{M^2+N}}$$
$$\times ln\left\{\frac{\sqrt{M^2+N}+M-v}{\sqrt{M^2+N}-M+v} \times \frac{\sqrt{M^2+N}-M}{\sqrt{M^2+N}+M}\right\} \qquad (35)$$

If V tends to zero, then the value above $\int (F_2-\bar{F}_2)\, dv$ tends to zero.

III. Process of Calculations and Examples

12) Process of calculation

i) a, σ, c, c_0, $\rho = \frac{C}{c_0}$, P_m, f, ω, G, $\Delta = \frac{\omega}{c_0}$, γ, i, A are given as data of a gun and explosive.

ii) The following should be calculated by auxiliary equations μ, z_0, P', P_0 δ, η', θ, θ', Θ, b.

If $\delta = 1.5$, $\eta = 1$ then $\eta' = 0.8$ and b, θ', and K can be calculated by equation (5).

iii) Auxiliary notations k, N, L, R, M, M_0, α, β, $\varepsilon = \frac{1}{\alpha} - 1$, $\frac{1}{2\alpha}$, $\frac{1}{2\alpha R}$ should be calculated and also a', b', and c' by equation (16).

iv) Construct θ', v curve by equation (21). This is nearly a straight line except when θ' is in the vicinity of 1. After all burnt, the v, θ curve can be drawn by equation (28).

Also the p, v curve shall be constructed by equation (22) after the curve of θ', v has been obtained. These two curves are shown in figures 1 and 3.

v) The position of the maximum pressure is obtained from equation (30) and the point of just burnt from equation (24).

13) Examples of calculation[7]

Example 1. Results of calculation and interior ballistic curves of a 12 inch (30 cm) gun.

To compare the author's methods of simple but exact solutions mentioned above with those of Charbonnier's ballistics, calculations are made on two kinds of propellant: cordite and tubite. The units used in this calculation are kg, dm, sec. (See table Ia, Fig. 1 and 2, and table Ib, Fig. 3 and 4)

a) Propellant used: MD Cordite (A case of all burnt in the bore)

i) Data of gun and projectile

$a = 3.082$ $\qquad c_0 = 295.0$

$\sigma = 7.459$ $\qquad P_0 = 32{,}000$

$c = 1.155$ $\qquad p = 400.6$

$\rho = \dfrac{C}{c_0} = 3.915$ $\qquad \gamma = 1.25$

ii) Auxiliary equations and constants of propellant

$\mu = 4.984$ $\qquad A = 0.0005342$

$P' = 647{,}600$ $\qquad i = 1.073$

$z_0 = 0.04941$ $\qquad f = 1{,}100{,}000$

$\Theta = 0.1775$ $\qquad \omega = 109.3$

iii) Auxiliary notations

$K = 1.784 \times 10^{-4}$ $\qquad N = 1.335 \times 10^{6}$

$\alpha = 0.8932$ $\qquad L = 3.703 \times 10^{-8}$

$R = 1.030$ $\qquad \beta = 0.6412$

$M = 4.700 \times 10^{3}$

b) Tubular powder grain

i) Gun and projectile are the same as the above.

ii) Auxiliary equation and constants of propellant

$\mu = 5.047$ $\qquad A = 0.0002671$

$P' = 760{,}700$ $\qquad i = 1.073$

$z_0 = 0.04207$ $\qquad f = 1{,}100{,}000$

$\Theta = 0.1686$ $\qquad \omega = 120.6$

iii) Auxiliary notations

$K = 9.036 \times 10^{-5}$ $\qquad N = 8.172 \times 10^{6}$

$\alpha = 0.1353$ $\qquad L = 5.147 \times 10^{-9}$

$R = 1.014$ $\qquad \beta = 0.2131$

$M = 1.754 \times 10^{4}$

Comparing these results above, the following conclusions can be extracted for cordite and tubite grains.

	The author's exact sol.		The author's simple sol.		Charbonnier's
P_m	356,800	>	335,900	<	347,700
$\bar{x}$	12.9	>	10.8	<	11.7
x	37.2	>	34.7	>	33.8
v	7744	<	7827	<	7984

For tubite or tubular grain

	The author's exact sol.		The author's simple sol.		Charbonnier's
P_m	250,600	>	249,520	<	252,500
$\bar{x}$	30.33	>	27.66	<	23.90
x	48.70	>	42.73	>	42.10
V	7458	<	7678	<	7718

Thus all explosives have the same tendency. Generally, the tendency is for the exact method of solution P_m to be the highest and initial velocity V the lowest, and the actual firing results are thought to be approximate to the accurate solution.

14) Example of calculation 2: A design of gun of 155mm calibre

Details of a gun and a projectile are given, it is required to find the highest velocity of shot attainable by selecting the most adequate size of cordiate powder or vivacity under a certain maximum pressure, 408,000kg/dm^2 (statical pressure is to be 1.2 times of maximum pressure by copper crusher 340,000kg/dm^2)

i) Details of gun and projectile

$a = 1.55$ $\qquad \sigma = 1.935$ $\qquad \rho = 5.478$

$c_0 = 35.5$ $\qquad C = 194.47$ $\qquad P_0 = 32{,}000$

$p=55.0$ $\quad i=1.049$ $\quad g=97.98$

$\gamma=1.25$ $\quad G=1$

For a propellant grain of Cordite, taking its force functions $f=1,020,000$ and $G=1$, $\eta=1$, calculations are proceeded for three kinds of powder charge $\omega_1=19,560$g, $\omega_2=18,500$g and $\omega_3=17,500$g. From a curve obtained through three values, assuming some value of vivacity $A_0=0.0006476$ and the corresponding weight of charge ω and initial velocity V can be obtained for the maximum pressure $P_m=408,000\text{kg}/dm^2$

	$A=0.0006476$		
ω	$\omega_1=19.56$	$\omega_2=18.56$	$\omega_3=17.50$
μ	$\mu_1=0.6935$	$\mu_2=0.6878$	$\mu_3=0.6825$
Δ	$\Delta_1=0.5509$		
P'	1.251×10	1.110×10	0.9918×10
z_0	0.02557	0.02883	0.03227
K	[4.06415]	[4.06060]	[4.05720]
$L\times10^8$	1.778	1.7766	1.7794
M_0	[3.8142]	[3.8109]	[3.8069]
M	[3.8086]	[3.8046]	[3.8069]
$N\times10^{-6}$	1.138	1.623	1.813
α	0.5115	0.4874	0.4653
R	1.0172	1.0198	1.0225
β	0.5057	0.4936	0.4821
$a'\times10^8$	2.742	2.697	2.656
$b'\times10^4$	4.6706	4.624	4.5806
c'	0.9478	0.9412	0.9342
v_1	8516	8571	8623
$\bar{v}$	3478	3351	3203
$bE(\bar{v})$	−0.2250	−0.0248	−0.03105
$bE(v_1)$	0.02491	0.02001	0.004925
θ_1'	0.1396	0.1202	0.1008
$\bar{\theta}'$	0.5640	0.5674	0.5644
$P_m\times10^{-3}$	442.6	379.25	324.81
Θ	0.09114	0.09661	0.10171
v	9072	8714	8445

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(6) This proof chiefly due to Professor T. Uno, Rigakuhakushi.

(7) Numerical calculation owed to Mr. Toshitada Miura.

Table I. (Example Ia, Cordite) See Fig 1 and 2.

	Yamaga's								Charbonnier's		
	exact solution ($G=1$)				simple solution ($G=1$)				$\beta=0{\cdot}5$		
	x	θ	$v\times10^{-3}$	$P\times10^{-3}$	x	θ	$v\times10^{-3}$	$P\times10^{-3}$	x	$v\times10^{-3}$	$P\times10^{-3}$
burning period	0	1	0	29.79	0	1	0	32.00	0	0	32.00
	0.98	0.965	0.50	128.3	—	—	—	—	—	—	—
	2.6	0.913	1.00	209.4	—	—	—	—	—	—	—
	6.7	0.804	2.00	317.4	6.0	0.80	2.000	3.063	6.00	2.033	316.0
	12.1	0.694	3.00	356.1	—	—	—	—	—	—	—
	19.4	—	4.00	337.6	17.6	0.60	4.000	312.0	20.0	4.247	318.1
	30.3	0.475	5.00	278.2	—	—	—	—	—	—	—
$\bar{x}$	12.9	0.680	3.133	356.8	10.8	0.695	3.014	335.9	11.7	3.092	347.7
x_1	37.2	0.424	5.465	242.0	34.7	0.415	5.463	228.4	33.8	5.466	246.8
all burnt	51.5	0.326	6.173	186.8	—	—	—	—	—	—	—
	73.9	0.252	6.912	135.4	80.0	0.235	7.164	112.8	80.0	7.300	119.3
muzzle	115.3	0.178	7.744	89.4	115.3	0.178	7.827	78.54	115.3	7.984	83.08

Table II. Example Ib, Tubular powder grain
(See Fig. 3 and 4)

	Yamaga's								Charbonnier's		
	exact solution ($G=0$)				simple solution ($G=0$)				$\beta=0$		
	x	θ	$v\times10^{-3}$	$P\times10^{-3}$	x	θ	$v\times10^{-3}$	$P\times10^{-3}$	x	$v\times10^{-3}$	$P\times10^{-3}$
burning period	0	1	0	32.00	0	1	0	32.00	0	0	32.0
	1.67	0.940	0.500	87.76	1.27	0.9486	0.500	94.70	—	—	—
	4.38	0.8612	1.000	136.7	4.30	0.8745	1.000	145.0	5.00	1.071	173.4
	10.80	0.7079	2.000	204.7	8.90	0.7243	2.000	211.9	14.00	2.743	239.6
	19.25	0.5761	3.000	240.2	16.24	0.5901	3.000	243.1	—	—	—
	29.08	0.4857	3.800	249.8	26.07	0.4729	4.023	249.0	33.0	4.635	247.5
$\bar{x}$	30.33	0.4632	4.023	250.6	27.66	0.4968	3.800	249.5	23.90	3.850	252.5
x_1	48.70	0.3496	5.300	222.7	42.73	0.3540	5.300	217.2	42.10	5.302	237.3
all burnt	66.2	0.2607	6.151	169.2	66.20	0.2607	6.443	145.2	65.00	6.433	163.3
muzzle	115.3	0.1750	7.458	98.13	115.3	0.1750	7.676	85.93	115.3	7.718	96.02

Fig. 1 $(v-\theta)$ and $(P-v)$ curves for powder charge of M D Cordfte propellant in a 30 cm gun

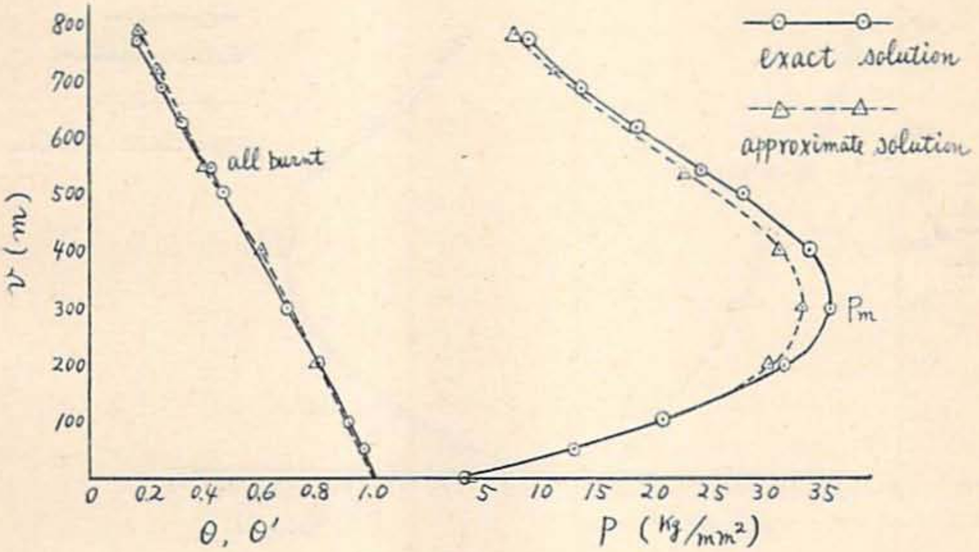

Fig. 2 Interior ballistic pressure-space curves for powder charge of M D Cordite propellant in a 30 cm gun

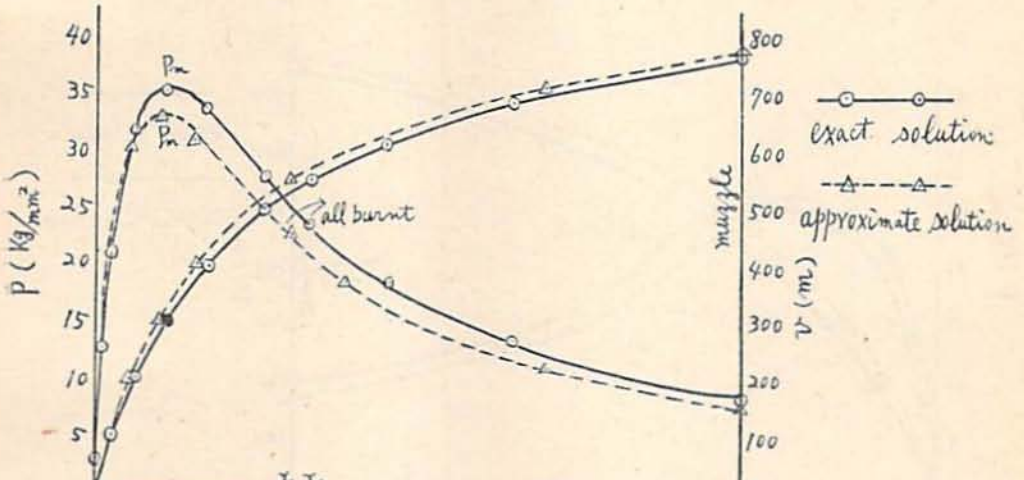

Fig. 3 $(v-\theta)$ and $(P-v)$ curves for powder charge of M D Tubite propellant in a 30 cm gun

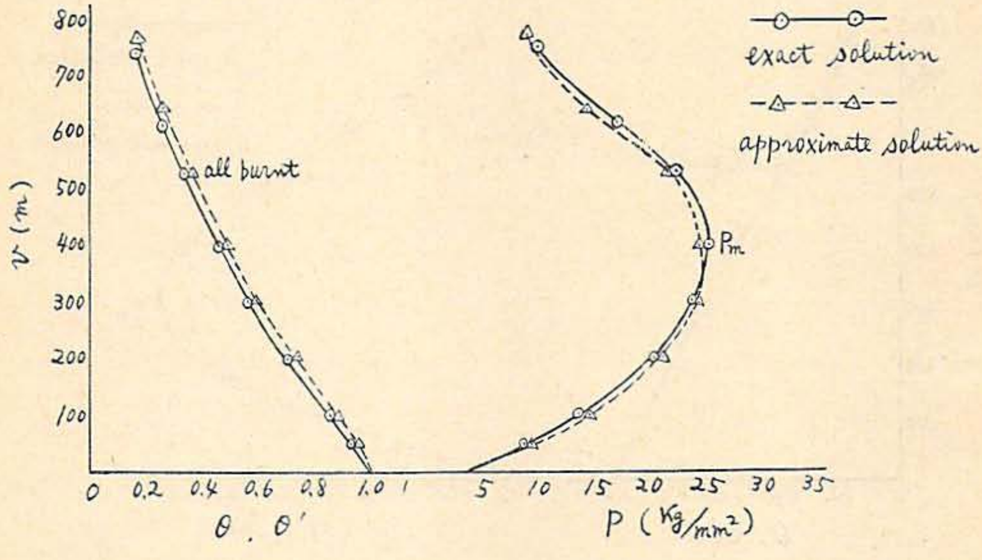

Fig. 4 Interior dallistic pressure-space curves for powder charge of M D Tubite propellant in a 30 cm gun