Letter

Burning and air resistance of fireworks stars

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Abstract

Burning and shot experiments have been carried out for examining the burning time and air resistance of burning fireworks stars. Burning time of various stars was determined using a high speed video camera. The burning times of No. 2 - No. 6 stars were 0.8 - 2.4 s. The mean linear burning rates were 3.0 - 6.0 mm·s^{-1} for 6 kinds of stars. The mean burning time was proportional to the mean diameter of same kind of star. The mean of relative standard deviation (standard deviation / mean value) of burning times of the stars was 0.051. The shot experiment of stars was done using a small mortar equipped with pressure sensors and the trajectory of the burning stars was observed by a high speed video camera. The relationship between the pressure in the mortar and initial velocity of the star was analyzed. The air resistance of a fired star was also discussed.

Keywords: Fireworks star, Burning time, Pressure in mortar, Initial velocity

1. Introduction

Fireworks star is one of the important elements of fireworks. It burns in appropriate time and produces flame, spark, glitter, light, sound, smoke, and so on. Burning stars move in the sky producing various shapes of artificial flowers of fire.

The stars of fireworks are used in two modes. In one mode, stars are put into a shell and expelled in the sky from the shell. In another mode, they are expelled into the sky directly from a mortar on the ground.

The motion of the stars expelled from the shell was studied by T.Shimizu[1), 2), 3)] and described in his book[4)].

The burning time and ballistics of burning stars were observed and analyzed in this paper.

2. Experimental

2.1 Materials

The stars used in the burning test and the shot experiment were supplied by Sunaga Fireworks Co. and their mean dimensions and standard deviations are listed in Table 1 along with the test results. The lifting charge was made by Nihon Kayaku Co..

2.2 Burning time of stars

Burning test of stars on a plate was carried out in a draft chamber. Burning test of stars in the sky was conducted in a baseball ground by expelling a star from a mortar of 25 mm inner diameter.

Burning time of stars on the plate and of stars expelled from the mortar was measured by high speed video cameras (FOR.A VFC-100SB, 250 frames·s^{-1} and Phantom VRI-V4.2, 600 frames·s^{-1}).

2.3 Ballistics of burning stars

The motion of stars in the sky was observed by the shot experiment of stars expelled from the mortar using the video cameras. The internal pressures of the mortar were measured by two pressure sensors (Kistler 60410 A), two amplifiers (Kistler 5011) and an oscilloscope (Sony Tektronix TDS3012) as shown in Fig. 1.

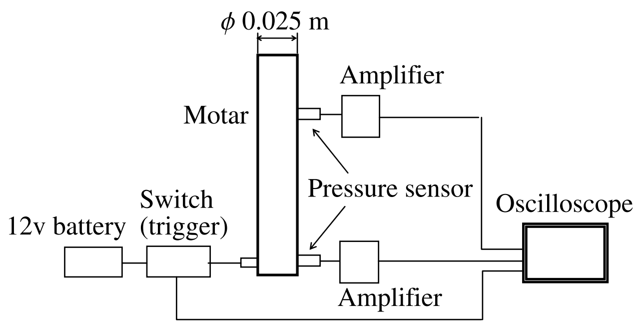

Fig. 1 Mortar and pressure measuring system.

Table 1 Mean and standard deviation of the dimension of stars and the burning time on a plate.

Star name	Shell No.	Numb. of exp.	Mass (g) mean	Mass (g) *std*	Diameter (mm) mean	Diameter (mm) *std*	Burning time (s) mean	Burning time (s) *std*
Pink peony	2	4	0.975	0.028	9.98	0.230	1.201	0.245
Red peony	2.5	5	1.148	0.021	10.69	0.170	1.005	0.067
Silver crown	3	4	1.716	0.036	11.84	0.130	1.602	0.069
Red peony	4	5	2.192	0.243	13.41	0.230	1.462	0.099
Violet peony	5	4	2.493	0.493	14.30	0.040	2.365	0.054
Silver peony	2	5	0.830	0.032	10.18	0.128	0.780	0.050
Silver peony	2.5	5	0.965	0.097	10.78	0.350	0.874	0.076
Silver crown	3	5	1.743	0.079	12.27	0.187	1.677	0.078
Silver peony	4	5	1.940	0.057	13.47	0.085	1.149	0.018
Silver peony	5	5	2.753	0.139	15.19	0.308	1.309	0.045
Silver peony	6	5	3.840	0.073	16.88	0.242	1.473	0.022
Blue peony	2	5	1.012	0.099	10.34	0.413	1.470	0.168
Blue peony	2.5	5	1.183	0.109	10.72	0.310	1.578	0.044
Blue peony	3	5	1.679	0.089	12.16	0.115	1.831	0.115

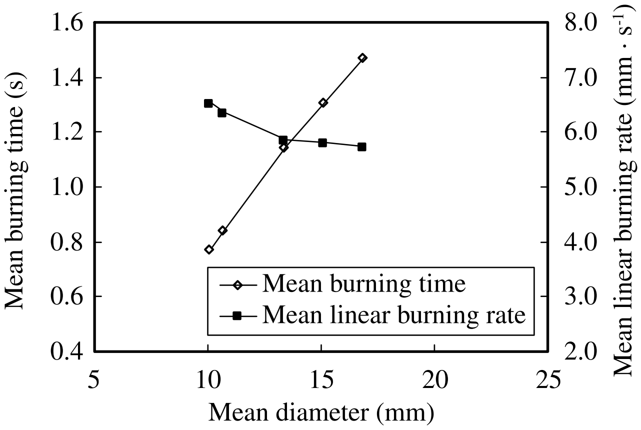

Fig. 2 Plots of mean burning time and linear burning rate vs. mean diameter of silver peony stars.

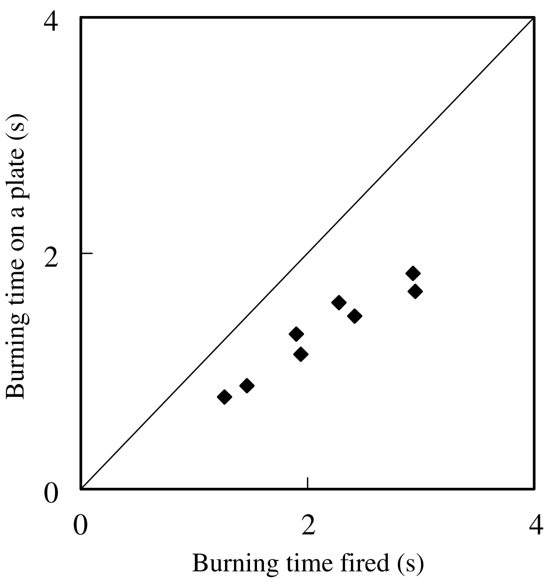

Fig. 3 Burning time of star.

Table 2 Kind of star and mean burning rate.

Kind of star	Mean linear burning rate ($mm \cdot s^{-1}$)
Blue peony	3.4
Pink peony	4.0
Red peony	5.0
Violet peony	3.0
Silver peony	6.0
Silver crown	3.7

3. Results and discussion

3.1 Burning time of stars

Mean and standard deviation of the dimension of stars and the burning time on a plate are listed in Table 1.

(1) Relationship between diameter of star and burning time

Figure 2 shows the plots of mean burning time and linear burning rate against mean diameter of the silver peony stars.

The burning time of the silver peony stars is nearly proportional to the diameter of the stars. But, the linear burning rate is not constant and decreases with increase of the diameter.

(2) The kind of star and the mean burning rate

The kind of star and the corresponding mean burning rate are listed in Table 2. The burning rate of stars is changed with the kind of stars.

(3) Scatter of observed data

Relative standard deviations of mass, diameter and burning time of the stars were calculated, in order to evaluate mutually the scatter of the observed data. Results are listed in Table 3.

Table 3 Relative standard deviations (*std.* / mean) of mass (*m*), diameter (*d*) and burning time (*t*).

	Star	$std.(m)\ m_{mean}^{-1}$	$std.(d)\ d_{mean}^{-1}$	$std.(t)\ t_{mean}^{-1}$
Blue peony	For No. 2 shell	0.098	0.040	0.114
	For No. 2.5 shell	0.092	0.029	0.028
	For No. 3 shell	0.053	0.018	0.063
Pink peony	For No. 2 shell	0.029	0.023	0.201
Red peony	For No. 2.5 shell	0.018	0.015	0.067
	For No. 4 shell	0.030	0.017	0.068
Violet peony	For No. 5 shell	0.014	0.003	0.023
Silver crown	For No. 3 shell	0.021	0.011	0.043
	For No. 3 shell	0.049	0.015	0.047
Silver peony	For No. 2 shell	0.039	0.013	0.064
	For No. 2.5 shell	0.097	0.032	0.087
	For No. 4 shell	0.029	0.006	0.016
	For No. 5 shell	0.050	0.020	0.034
	For No. 6 shell	0.019	0.014	0.015
Mean		0.053	0.018	0.051

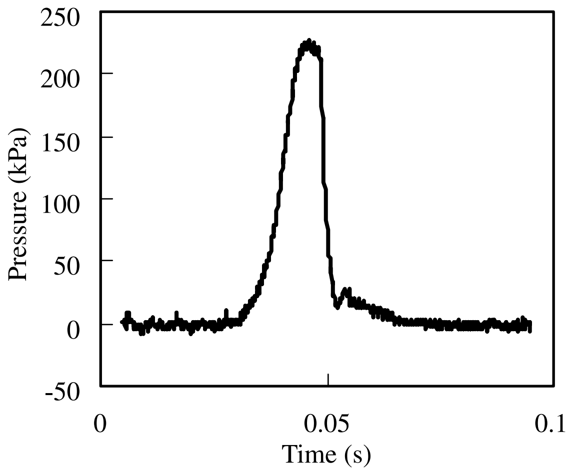

Fig. 4 Pressure profile in the mortar, shot 10.

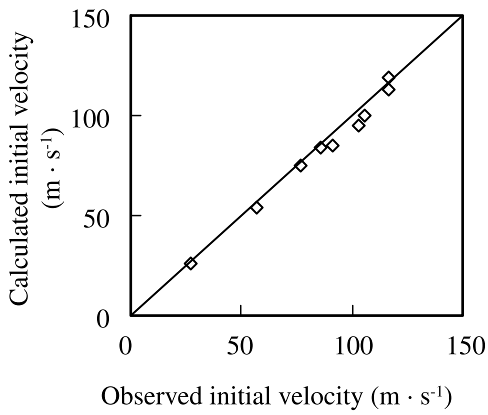

Fig. 5 Plot of calculated initial velocity vs. observed one.

Table 4 Burning time of star fired into the sky and on a plate.

Name	Shell no	Burning time (s) Fired	On a plate
Blue peony	for 2.5	2.272	1.578
Blue peony	for 3	2.932	1.831
Silver peony	for 2	1.260	0.780
Silver peony	for 2.5	1.468	0.874
Silver peony	for 4	1.932	1.149
Silver peony	for 5	1.900	1.309
Silver peony	for 6	2.420	1.473
Silver crown	for 3	2.944	1.677

*The mean value

The mass of star is proportional to the third power of the diameter. Therefore, the standard deviation of the mass is proportional to three times of that of the diameter. The mean of relative standard deviation of the mass was 0.053 and 2.9 times of that of the diameter 0.018.

The scatter of the burning time may be divided into that of the diameter of stars and other factors. The mean relative standard deviation of burning time was 0.051 and that of the diameter was 0.018. The difference is 0.033 and this may be attributable to the scatters of both combustion phenomenon and time measurement.

(4) Burning time of moving star

Burning time of the stars fired into the sky and on a plate was listed and shown, in Table 4 and Fig. 3, respectively.

Burning time of the star fired into the sky was longer than that on a plate. This may be attributable to the cooling effect of airflow on the temperature of the flame of the star fired into the sky.

Table 5 Dimension of star, mass of lifting charge and initial velocity of fired star.

Shot No.	Mass of star (g)	Diameter of star (mm)	Mass of lifting charge (g)	Initial velocity ($m \cdot s^{-1}$) a*	b*
Shot 1	9.521	22.356	1.0	87	
Shot 2	9.517	22.701	0.4	27	26
Shot 3	9.065	22.318	0.7	57	54
Shot 4	9.244	22.166	2.0	117	113
Shot 5	8.346	21.756	1.5	103	95
Shot 6	8.924	22.410	1.5	106	100
Shot 7	8.586	22.511	2.0	117	119
Shot 8	8.846	22.626	1.5	86	84
Shot 9	8.308	22.353	1.25	77	75
Shot 10	8.475	22.045	1.75	91	85

a* Initial velocity measured by a camera
b* Initial velocity calculated from pressure profile in the mortar

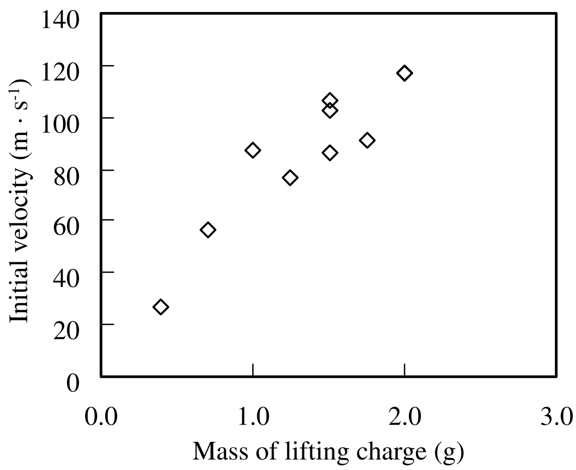

Fig. 6 Plot of observed initial velocity vs. mass of lifting charge.

3.2 Ballistics of burning stars

(1) Relationship between internal pressure of the mortar and initial velocity of the burning star

An example of pressure profiles in the mortar during the shot of the star is shown in Fig. 4.

The motion of a star in the mortar was analyzed by a personal computer using the same method as before[5]. Results are listed in Table 5. Initial velocity of a shell can be estimated by calculation from the pressure profile in the mortar. As shown in Fig. 5, the accuracy of the estimation of initial velocity from the pressure profile is rather good.

Plot of the initial velocity of a star against the mass of lifting charge is shown in Fig. 6. The initial velocity increases with the mass of lifting charge. But, scatter is rather large. This may come from the irregular burning of the lifting charge.

(2) External ballistics of a star

The motion of a burning star fired vertically in the sky is expressed by following equations:

$$\frac{du}{dt} = -a^2 - b^2u^2 \qquad (1)$$

Here, $a^2 = g$, $b^2 = \frac{3\rho_f}{4\rho D} C_D$,

g, ρ_f, ρ, D, and C_D are acceleration of gravity, air density, density of the star, diameter of the star, air resistance, and drag coefficient, respectively.

Integral for time t:

$$t = t_0 - \frac{1}{ab}\left(\arctan(\frac{b}{a}u) - \arctan(\frac{b}{a}u_0) \right) \qquad (2)$$

That is:

$$u = \frac{a}{b}\tan\left(\arctan(\frac{b}{a}u_0) - ab(t - t_0) \right) \qquad (3)$$

Integral for moved height h:

$$h = h_0 - \frac{1}{2b^2}\ln(\frac{a^2 + b^2u^2}{a^2 + b^2u_0^2}) \qquad (4)$$

Here, t_0, h_0, and u_0 are initial time, initial height, and initial velocity, respectively.

After the star attained highest height, descent of the star begins.

Equation of free falling motion of the star:

$$\frac{du}{dt} = -a^2 + b^2u^2 \qquad (5)$$

Integral for time:

$$t = t_1 - \frac{1}{2ab}\ln\left(\frac{a + bu}{a - bu} \right) \qquad (6)$$

That is:

$$u = -\frac{a}{b}\tanh\left(ab(t - t_1)\right) \quad (7)$$

Integral for fall height h:

$$h = h_1 + \frac{1}{2b^2}\ln\left(\frac{a^2 - b^2u^2}{a^2}\right) \quad (8)$$

Here, t_I and h_I are time of flight to altitude attained and altitude attained, respectively.

The shot experiment of a star was carried out using a following star for No.10 shell: mass 0.00848 kg, diameter (D) 0.0220 m and density (ρ) 1510 kg·m^{-3}. Other parameters: the acceleration of gravity (g) 9.8 m·s^{-2}, and air density (ρ) 1.205 kg·m^{-3}.

The observed initial velocity of the star was 91 m·s^{-2}. The observed height of the star against time was plotted in Fig. 7.

Assuming C_D of 0.0, 0.3, 0.5 and 0.7, the height of the star was calculated for time and plotted in Fig. 7. In this case, C_D = 0.5 was best fit to the observed value.

Above analysis neglected the change of diameter of the star with time. The more accurate analysis will be conducted in the future work.

4. Conclusions

(1) The mean linear burning rates of the stars on a plate were from 3.0 to 6.0 mm·s^{-1}.

(2) The burning time of the stars fired into the sky was 1.6 times longer than that on a plate.

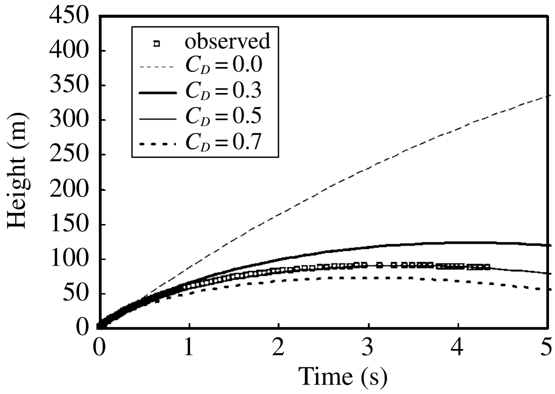

Fig. 7 Plot of height vs. time for a fired star.

(3) In the shot of a star from a mortar, initial velocity of the star was calculated from the internal pressure profile.

(4) The correlation of the mass of lifting charge and the initial velocity of fired star was not good.

References

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煙火星の燃焼と空気抵抗

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煙火星の燃焼時間と燃焼中の星の空気抵抗を検討するために，星の燃焼実験と打揚実験を行った。種々の星の燃焼時間を高速度ビデオカメラを用いて測定した。２号玉用から6号玉用の星の燃焼時間は0.8-2.4 s であった。6種の星の平均線燃焼速度は3.0-6.0 mm･s^{-1}であった。星の燃焼時間の相対標準偏差（標準偏差/平均値）の平均は0.051であった。圧力センサーを備えた小さな打揚筒を用いて星の打揚実験を行い，高速度ビデオカメラを用いて燃焼星の軌跡を観測した。打揚筒内の圧力と星の初期速度との関係を解析した。発射された星の空気抵抗についても考察した。

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